CENTRIFUGATION OF RAW SEWAGE

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CENTRIFUGATION OF RAW SEWAGE

By

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LIST OF FIGURES

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ABSTRACT

Centrifugation of raw sewage as an alternative to primary gravity sedimentation was investigated, and its potential use as a unit operation in an advanced physical treatment system was evaluated. The centrifuges used were the disc type and the solid bowl type with flowrates in the range 2.5 to 19 l/min (0.55 to 4.2 Igpm). Values of solids removal efficiencies as high as 98% were observed for the disc centrifuge. The solid bowl centrifuge produced sludge cakes containing an average of 70% solids. The suspended solids removal efficiency for the solid bowl unit was observed to increase with increasing pond depths and decrease with increasing flowrates and scroll angular velocity differentials. The disc unit displayed higher efficiencies at lower flowrates, shorter cycle times and full desludging operation. The average values for BOD removal efficiencies were 23% and 49% for the solid bowl and disc units respectively. Phosphate removal efficiencies averaged 25% and 28% for the solid bowl and disc units respectively. The variations in the disc centrifuge sludge concentrations were analyzed by deriving and using a simple mathematical model. The maximum error in using this model was ±30% for the particular machine and example analyzed.

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PROJECT STAFF

The following staff of the Applied Sciences Section, were instrumental in the production of this report.

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ACKNOWLEDGEMENT

The author thanks the Sharples - Stokes Division of the Penwalt Corporation and Mr. J.R. Townsend, Jr. for providing the model P-600 Super-D-Canter Solid Bowl Centrifuge, and the Westfalia SAMN-3 Disc Centrifuge.

1.0 INTRODUCTION

Within the past few years there has been considerable interest in the application of centrifuges in the wastewater treatment field. New technology in performance and sizing along with a concerted effort by industry in the wastewater treatment field has resulted in centrifugation becoming an efficient unit operation for solids dewatering of primary, secondary and combined wastewater sludges.

Most existing wastewater treatment systems utilize primary gravity sedimentation to remove settleable solids. Detention periods for suspended material may vary from ½ hour to 1.5 hours for a well-operated conventional primary sedimentation basin, while a centrifuge, processing the same slurry may produce an effluent of equal or better quality in 1 or 2 minutes(1). Thus, it is possible that centrifugation of raw sewage could reduce required detention times and eliminate the use of large space-occupying clarifiers with 85 to 98 percent recovery of sludge, and produce sludge cakes containing 15 to 25 percent solids. These values may be compared to the 40 to 70% removal efficiences resulting from primary sedimentation facilities producing sludges of less than 7% solids.

Studies on centrifugation have been performed previously(1) on activated sludge, combined primary and secondary digested sludge(2), sulphite papermill primary

sludge, lime sludge [3] and digested sludge, but no reported studies have involved raw sewage.

In June 1972, a preliminary feasibility study carried out by the Research Branch demonstrated the centrifuge to be a viable alternative to primary gravity sedimentation for both sewage clarification and sludge recovery.

The purpose of this present study is to conduct tests on equipment of a larger scale and evaluate the effect of the various operating parameters on solids removal efficiencies and cake quality.

2.0 DESCRIPTION OF CENTRIFUGE AND THEIR OPERATION

2.1 Centrifuge Types

Centrifuges for wastewater treatment are generally categorized into three types; solid bowl, disc and basket. Each performs one of three functions; clarification, classification or separation. Clarification is the removal of solids from a slurry where either the supernatant liquid or the concentrated solids are to be recovered. Classification is the removal of solid particles of a specific size. Separation is the process of splitting a liquid into two parts, a light phase and a heavy phase. For the above processes to be feasible, a difference in phase density must exist. In this study we are primarily interested in clarification for solids removal from raw sewage. The centrifuge types investigated were the solid bowl and the disc configurations.

2.2 The Solid Bowl Centrifuge

According to a review of centrifuges by Smith [4], solid bowl models are generally considered to have the best combination of clarification and dewatering properties for most wastewater treatment sludges. As shown in Figure 1 it is of the cylindrical type with a conical dewatering section and supported between two bearings. At the end opposite the dewatering section is a dam to regulate the liquid depth in the bowl.

Sludge is introduced through the hollow shaft and enters the annular space within the bowl. The centrifugal force causes the sludge to move to the outer periphery

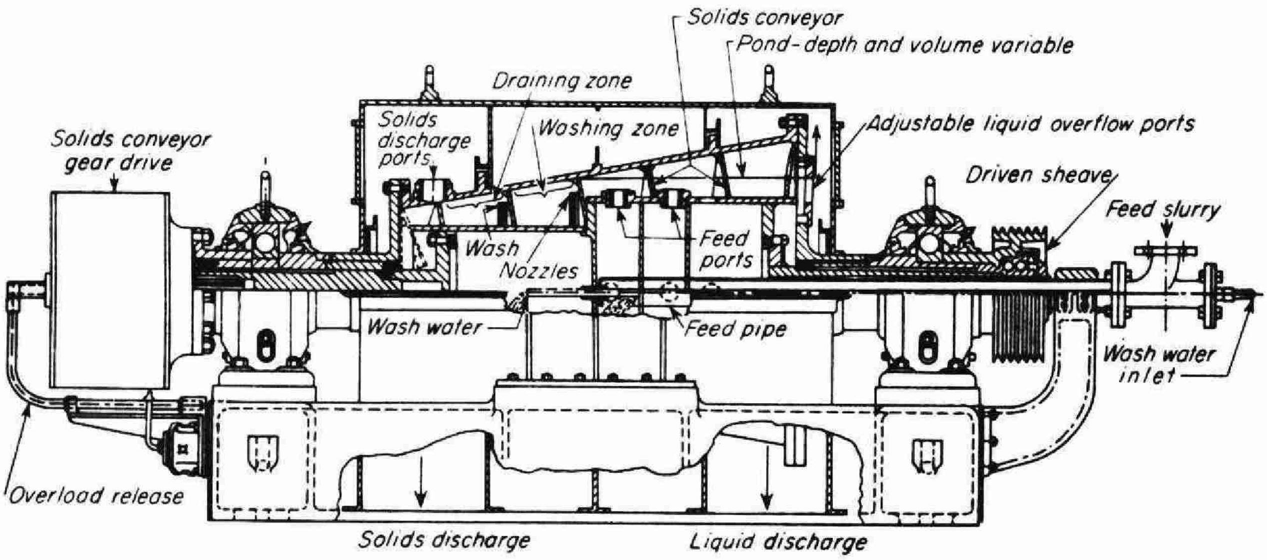


Figure 1. Solid Bowl Centrifuge (Bird Machine Co.)

of the bowl and consolidate on the bowl wall. The pond depth is controlled by the regulating dam, over which clarified effluent is removed.

As the liquid flows through the conical section towards the dam, progressively finer solids settle on the bowl wall. The solids are removed to the discharge ports by a helical screw, or scroll, which rotates at a speed slightly different from that of the bowl. Because of the opposite direction of movement of the two phases in the bowl, the solid bowl centrifuge is sometimes referred to as a countercurrent machine.

The operation of the solid bowl centrifuge is dependent on a number of parameters:

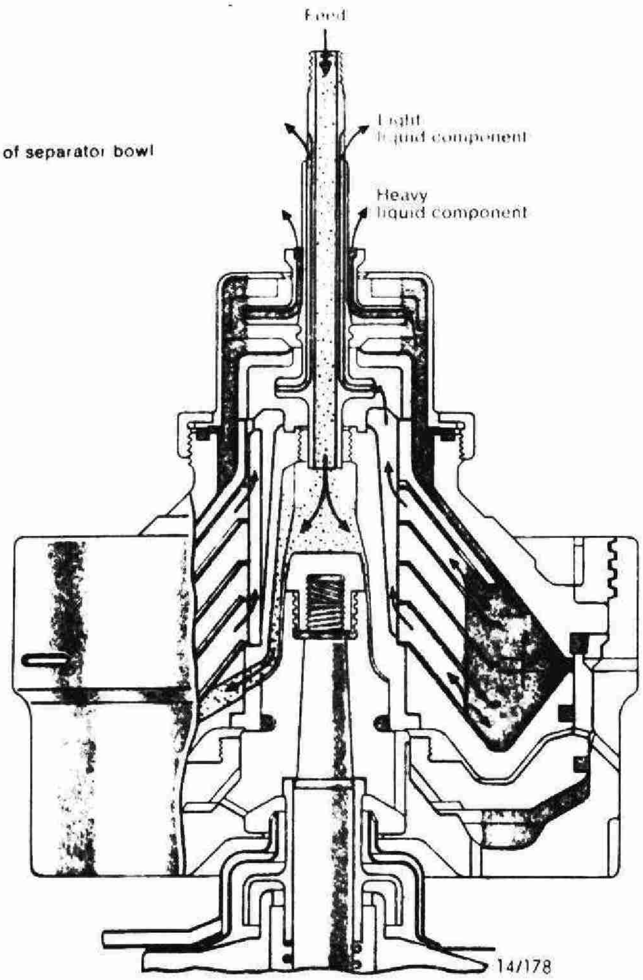
(1) feed flow rate
(2) centrifugal force (diameter and rotational speed)
(3) liquid level (pond or pool depth)
(4) speed differential between the bowl and conveyor.

The effect of these variables were investigated in this study, and because they are not independent of each other, an optimum setting of each may not produce the desired solids removal or the greatest degree of cake dryness.

2.3 Disc Centrifuge

In the disc centrifuge the clarifying section consists of a number of conical discs stacked one upon another as shown in Figure 2. Each disc acts as a separate centrifuge of very low capacity. The incoming liquid mixture

Cross section of separator bowl



Flow pattern of the product in the disc set

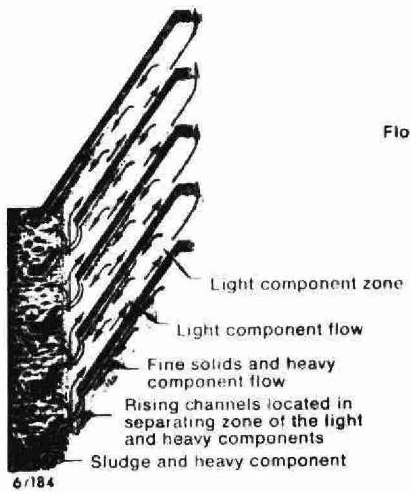


Figure 2. Disc Centrifuge.
(Courtesy Sharples-Stokes
Division, Penwalt Corp.)

enters the separation space through the holes in the distributor bases and flows into the rising channels from which it passes into the narrow spaces between the discs. The disc spacing has a direct effect on the performance of the centrifuge in solids recovery. The settled solids are removed as they slide down the underside of the upper disc, and pass to the outer perimeter of the bowl. Meanwhile, the liquid passes inward to the upper end of the bowl and is pumped out by the centrifugal pump. After a prescribed operating time, the feed valve closes, and an operating water valve opens for a few seconds, filling up a chamber. In doing so, it rapidly depresses a sliding piston thus opening the discharge ports and allowing the solids to be ejected. After the water supply stops, the bowl closes automatically and the feed valve is reopened. The sludge discharge operation is done with the bowl rotating at full speed. In addition the bowl may be completely discharged for a full desludge at the end of a cycle or only partly discharged by opening the sliding piston for a very short time interval. The latter operation is termed a partial discharge.

The performance parameters for the disc centrifuge are:

(1) the centrifugal force,

(2) the disc spacing,

(3) the feed rate,

(4) the desludge quantity,

(5) the cycle time.

The disc spacing is a significant parameter because it determines whether or not solids will be removed from the slurry phase. A very minimum spacing between disc is not always advantageous because plugging can occur.

2.4 Comparison of Centrifuges

In general, centrifuges may be considered to be high intensity settling basins. The rate of settling in a normal sedimentation tank is limited by the value of the gravitational acceleration, g. In centrifuge operations the acceleration is $r\omega^2$, where r is the distance of a particle from the axis of rotation and ω is the angular velocity. Values for $r\omega^2$ may be 1500 to 4000g for solid bowl machines.

Ambler [5,6] evaluated centrifuges on the basis of substitution of $r\omega^2$ for g to determine an index of centrifuge size, "Σ".

Although the values of Σ are useful in comparing design properties of various machines, this approach is based on the sedimentation of discrete and individual particles rather than the hindered settling of the sludge mass. Moreover, the effect of scrolling is not taken into account in the analysis of the solid bowl machine.

Therefore, the centrifuge variables must be studied in the light of the whole clarification operation with the values of solids removal and cake consistencies as indices of performance.

3.0 EXPERIMENTAL EQUIPMENT AND PROCEDURE

3.1 General Description of Experimental System.

3.1.1 Solid Bowl Centrifuge

The arrangement of the apparatus for testing the solid bowl centrifuge is shown in the flow diagram of Figure 3.

Raw sewage was pumped by a submersible pump from a holding tank through a rotameter into the centrifuge which was set at the desired values of flowrates, pond depth and scroll differential angular velocity. The centrate then flowed to a set of holding tanks to allow the removal of air bubbles from the supernatant solution. It then passed through 5 micron and 1 micron cartridge filters connected in series. Turbidity was monitored continuously before and after the filters. At the same time the sludge cake was continuously removed from the centrifuge.

3.1.2 Disc Centrifuge

The apparatus for the disc centrifuge experiments was essentially the same as for the solid bowl except that the raw sewage was screened before entering the disc centrifuge in order to prevent large particles from plugging the disc spaces. The flow diagram is shown in Figure 4. The cartridge filters used were also of the 5 micron and 1 micron pore sizes.

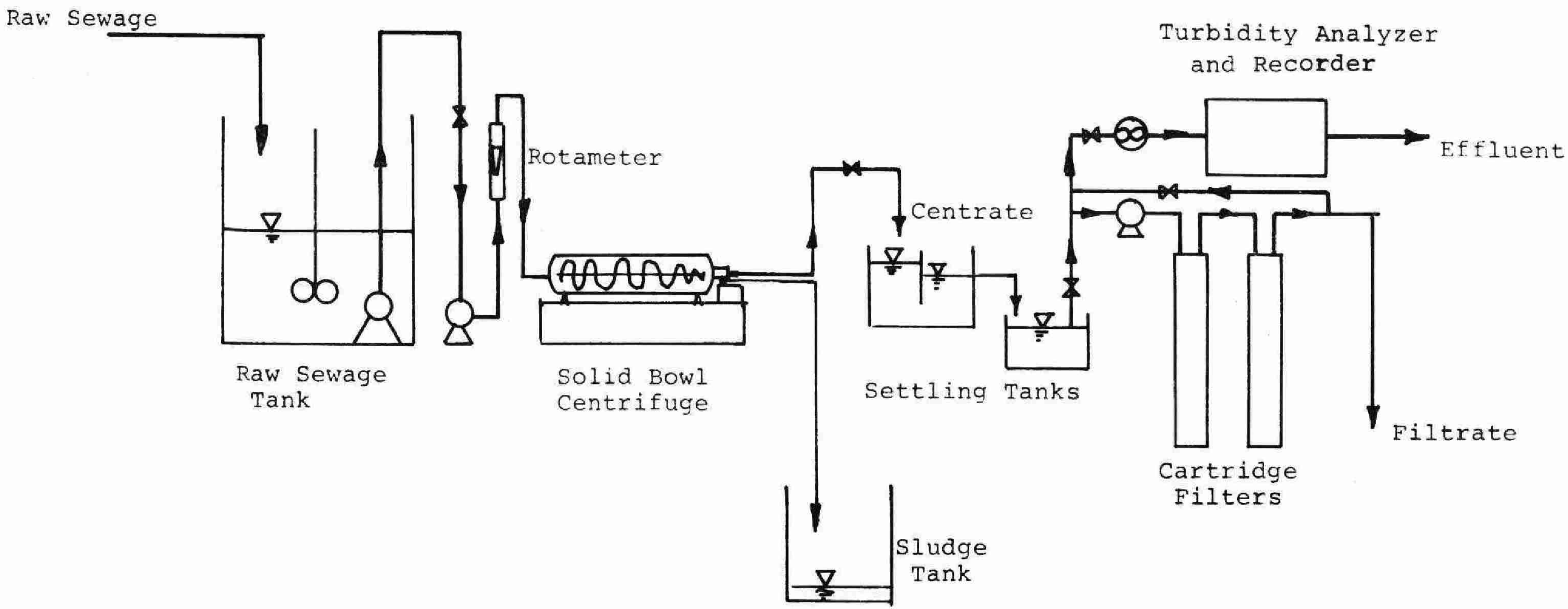


Figure 3. Flow Diagram for Solid Bowl Centrifuge.

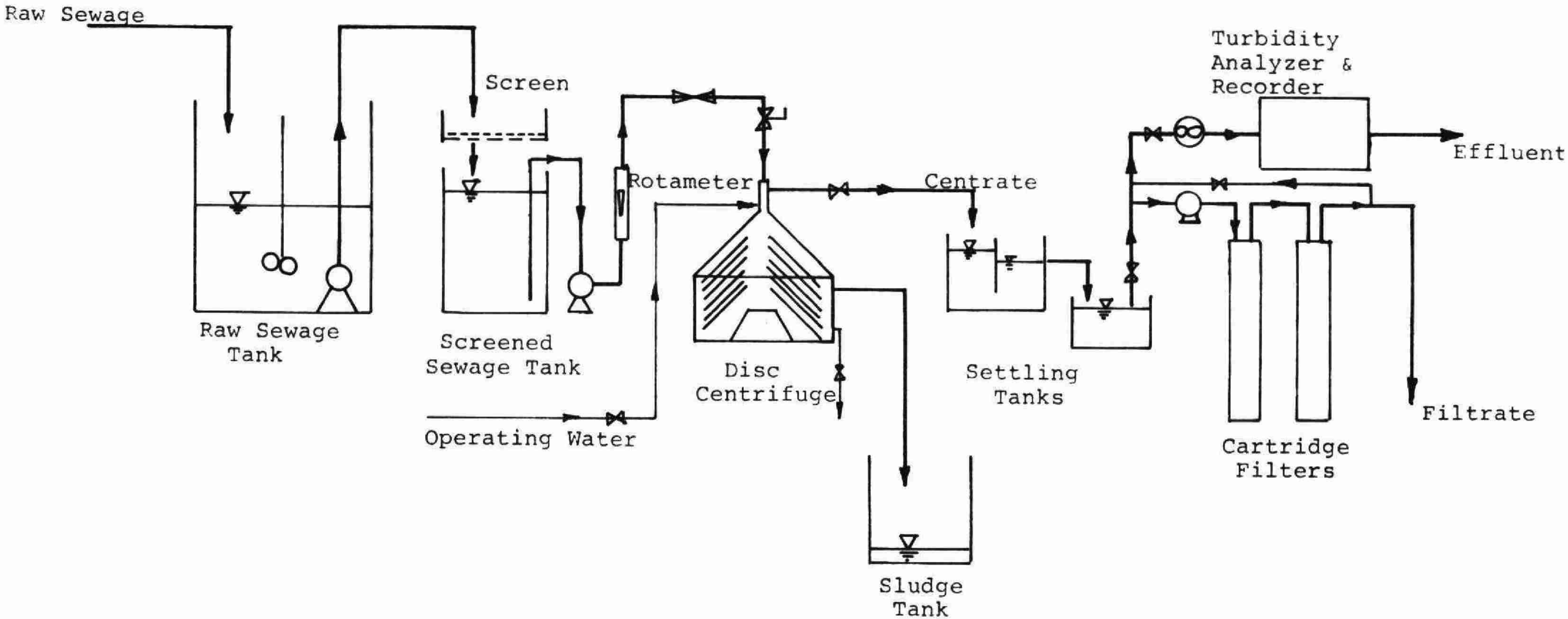


Figure 4. Flow Diagram for Disc Centrifuge.

3.2 Centrifuge Characteristics and Experimental Parameters

The Solid Bowl Centrifuge used in this project was a Sharples Model P-600 Super D-Canter and the disc centrifuge was a Westfalia Model SAMN-3, both from the Sharples Stokes Division of the Pennwalt Corporation. The operating parameters for this experiment are listed in Table 1.

All tests were conducted using raw sewage from the community of Rexdale, Ontario.

3.3 Sample Analysis

The major variable analyzed was the concentration of suspended solids in the feed, centrate and sludge. The efficiency of the centrifuge was evaluated in terms of the suspended solids removal and the solids concentration in the sludge or cake. The values for suspended solids were determined from total solids and dissolved solids analyses according to "Standard Methods"[7]. Percent turbidity of the centrate and filtrate was monitored throughout the experiments using a photometric turbidity analyzer.

Other variables analyzed were;

5-day Biochemical Oxygen Demand (BOD_5)

Total and Soluble Phosphorus as P

Total Kjeldahl Nitrogen as N

Nitrite as N

Nitrate as N

pH

TABLE 1

Operating Parameters for Solid Bowl and Disc Centrifuges

Parameter	Solid Bowl Centrifuge		Disc Centrifuge	
Feed Rate	3.0 l/min to 11.37 l/min	(0.66 Igal/ min to 2.5 Igal/min)	2.47 l/min to 18.95 l/min.	(0.54 Igal/ min to 4.2 Igal/min)
Bowl RPM	4900		6700	
ΔRPM	12 - 38			
Pond Depth	1 - 4 setting positions			
Total Desludge Cycle Times			10 and 20 min.	
Partial Desludge Cycle Times *			10, 20 & 30 min.	

* A cycle is defined as the operating time between the closing and the opening of the desludge valve.

Alkalinity

Conductivity

Formazin Turbidity

Suspended solids data were obtained for each change in operating conditions while the other sets of analyses were performed on the first and last samples of each day's run.

3.4 Performance Calculations

For evaluation of centrifuge performance, the percent solids removal was calculated from the relation:

$$\% \text{ solids removal} = \eta_{ss} = \frac{C_{ss}^{R} - C_{ss}^{C}}{C_{ss}^{C}} \times 100\% \quad \ldots.(1)$$

where $C_{ss}^{R,C}$ are the concentration of suspended solids in raw sewage and the centrate, mg/l, η_{ss} is the suspended solid removal efficiency, %.

In terms of the measured quantities,

$$\eta_{ss} = \frac{(C_{TS}^{R} - C_{DS}^{R}) - (C_{TS}^{C} - C_{DS}^{C})}{C_{TS}^{R} - C_{DS}^{R}} \times 100\% \quad \ldots\ldots.(2)$$

and at steady state it may be assumed that

$$C_{DS}^{R} = C_{DS}^{C},$$

$$\text{then} \quad \eta_{ss} = \frac{C_{TS}{}^{R} - C_{TS}{}^{C}}{C_{TS}{}^{R} - C_{DS}{}^{R}} \times 100\% \qquad \ldots\ldots\ldots(3)$$

where $C_{TS}{}^{R,C}$ are the total solids concentration in the feed and centrate, $C_{DS}{}^{R,C}$ are the dissolved solids in the feed and centrate.

Equation (3) was also used on the data from cartridge filters using the values of SS in the centrate and the filtrate. The percent solids in the sludge was obtained from the relation;

$$\%\ \text{Sludge} = \frac{\text{weight of dried sludge}}{\text{net weight of sludge}} \qquad \ldots\ldots\ldots(4)$$

4.0 DISCUSSION OF RESULTS

4.1 Operation of Equipment

No major operational problems were encountered with either centrifuge. Clarification of raw sewage was easily achieved without the use of any chemical coagulants regardless of the sewage strength. In addition, the operation was odour free.

The centrate produced by the solid bowl machine appeared to be of a clearer nature than the feed indicating a definite removal of suspended material. The sludge cake from this unit was of a semi-dry fibrous nature. On a few occasions, particularly at the beginning of a run, a watery sludge was produced which gradually thickened as the operation proceeded.

The centrate from the disc centrifuge was clearer than that from the solid bowl unit, but the sludge produced was a concentrated non-settling suspension of a dark brown appearance. The average volume of sludge produced from a full desludge operation was 6.2 l (1.4 Igal) while the partial desludge volume varied from 200 ml to 3000 ml during constant conditions of flow and cycle times. The average partial desludge volume was 1,250 ml or approximately 20% of the full discharge value.

No difficulties with plugging of the disc were encountered. This was probably prevented by the prescreening of the raw sewage.

4.2 Evaluation of the Solid Bowl Centrifuge

4.2.1 Suspended Solids Removal

The optimization of the centrifugation process involves a complete understanding of the behaviour of the system when the process variables are changed within their desired range of values. One of the indices of performance characterizing a centrifuge is the solids removal efficiency relative to the feed concentration. For a fixed feed concentration the solids removal efficiency is dependent on the variation of the suspended solids in the centrate. To observe the behaviour of the centrate concentration with changes in the operating parameters, the experimental data have been plotted in terms of the centrate suspended solids concentration, for variations in feed concentration, feed flowrate, pond depth and differential scrolling RPM.

Figure 5 shows the variation of the centrate suspended solids with feed rate for a set of typical operating conditions, namely, ΔRPM = 20, pond depth at position 2 and two averaged values of feed concentration, 585 mg/l and 441 mg/l of suspended solids. The data for the runs are listed in Table 2.

Both lines drawn through the data display positive slopes for two different intercepts. These plots indicate that the centrate suspended solids (SS) is increasing concommitantly with increases in flowrate of sewage feed. The increasing flowrate implies a decrease in residence

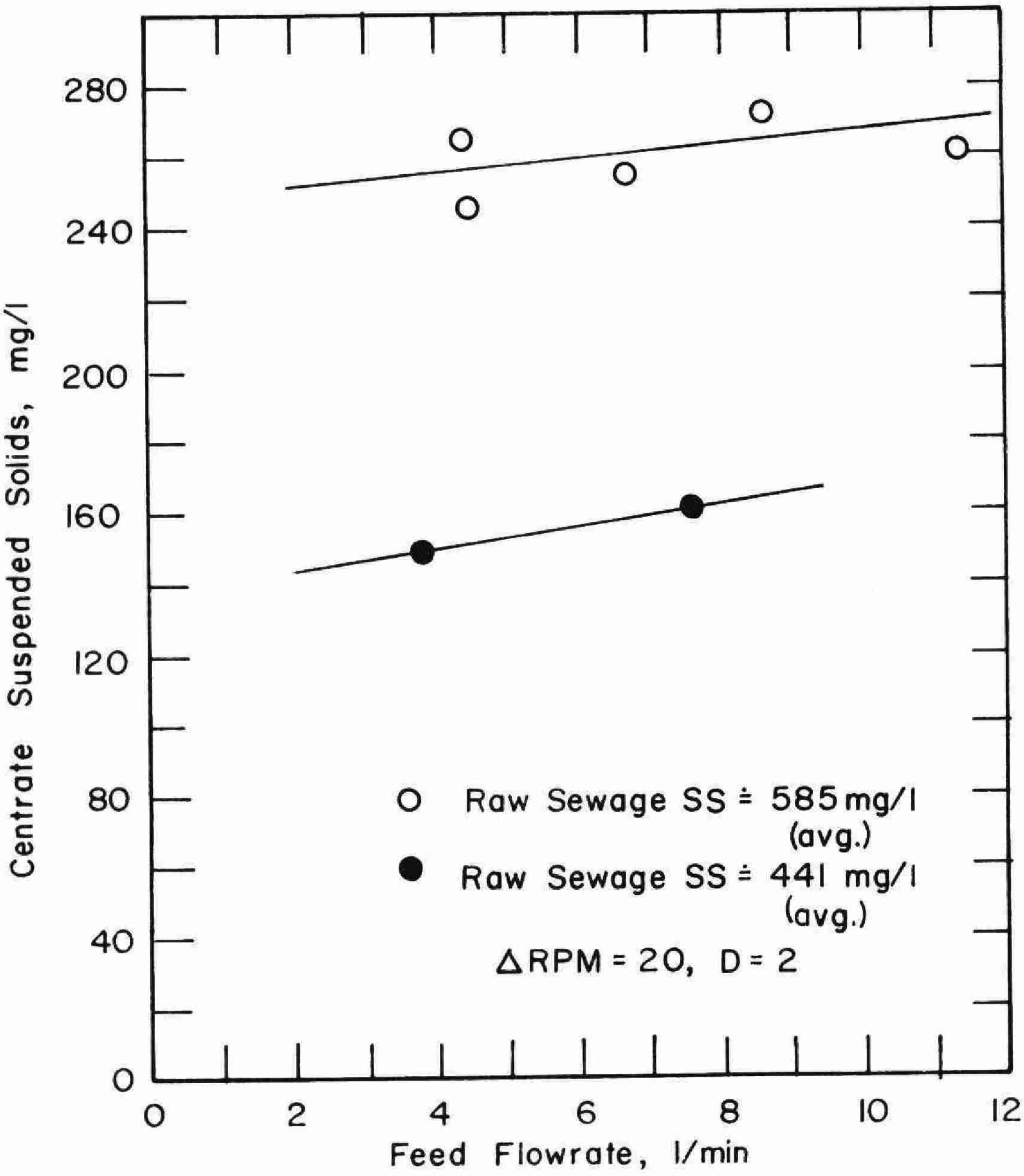


Figure 5. Variation of Centrate Suspended Solids with Feed Flowrates for Solid Bowl Centrifuge, for two Suspended Solids Feed Conditions.

TABLE 2

CENTRATE SUSPENDED SOLIDS

Run #	Flowrate USGM	Flowrate l/min	ΔRPM	D	RAW TS	DS	CENTRATE TS	RAW SS	CENTRATE SS mg/l
24	2.0	7.58	20	2	940	444	605	496	161
25	1.0	3.79	20	2	931	444	593	387	149
26*	3.0	11.37	20	2	1210	672	933	538	261
27	2.25	8.54	20	2	1250	672	944	578.	272
28	1.75	6.64	20	2	1230	640	895	590	255
29	1.15	4.36	20	2	1245	619	884	626	265
30	0.65	4.46	20	2	1215	619	864	596	245
36	1.42	5.29	26	4	1076	631	854	445	223
37	1.42	5.29	26	3	1050	638	888	412	250
38	1.42	5.29	26	2	1099	645	901	454	256
39	1.42	5.29	26	1	1066	652	938	414	286
40	1.40	5.29	20	1	1260	739	1028	521	289
41	1.40	5.29	20	2	1203	744	940	459	196
42	1.40	5.29	20	3	1232	749	911	438	162
43	1.40	5.29	20	4	1156	755	863	401	108

* Indication of different feed sewage than used in runs 24 and 25.

time in the bowl, giving the solids within less time to settle to the bowl periphery. Consequently, less solids are removed from the stream and the centrate concentration increases in comparison to the values at the lower rates.

Although there is insufficient data for a more complete comparison the two lines of Figure 5 appear to be approximately parallel. The existence of almost similar slopes indicates that the rates of change of suspended solids concentrations with respect to flow rates (or residence times) are approximately equal for both feed concentrations and that this rate is a function of the machine properties, especially the screw differential RPM and the pond depth.

4.2.2 Pond Depth and Differential Angular Velocity

Increases in pond depth have the effect of increasing the liquid residence time within the bowl. In turn, increasing the particle retention time decreases the concentration of suspended solids in the centrate. Figure 6 shows this to be the case when the centrate concentration is plotted against the pond depth. The data in this plot are for approximately the same feed concentration but different angular velocity differentials.

As the differential angular velocity, ΔRPM, is increased, the SS concentration in the centrate also increases. This effect is again due to the reduction in

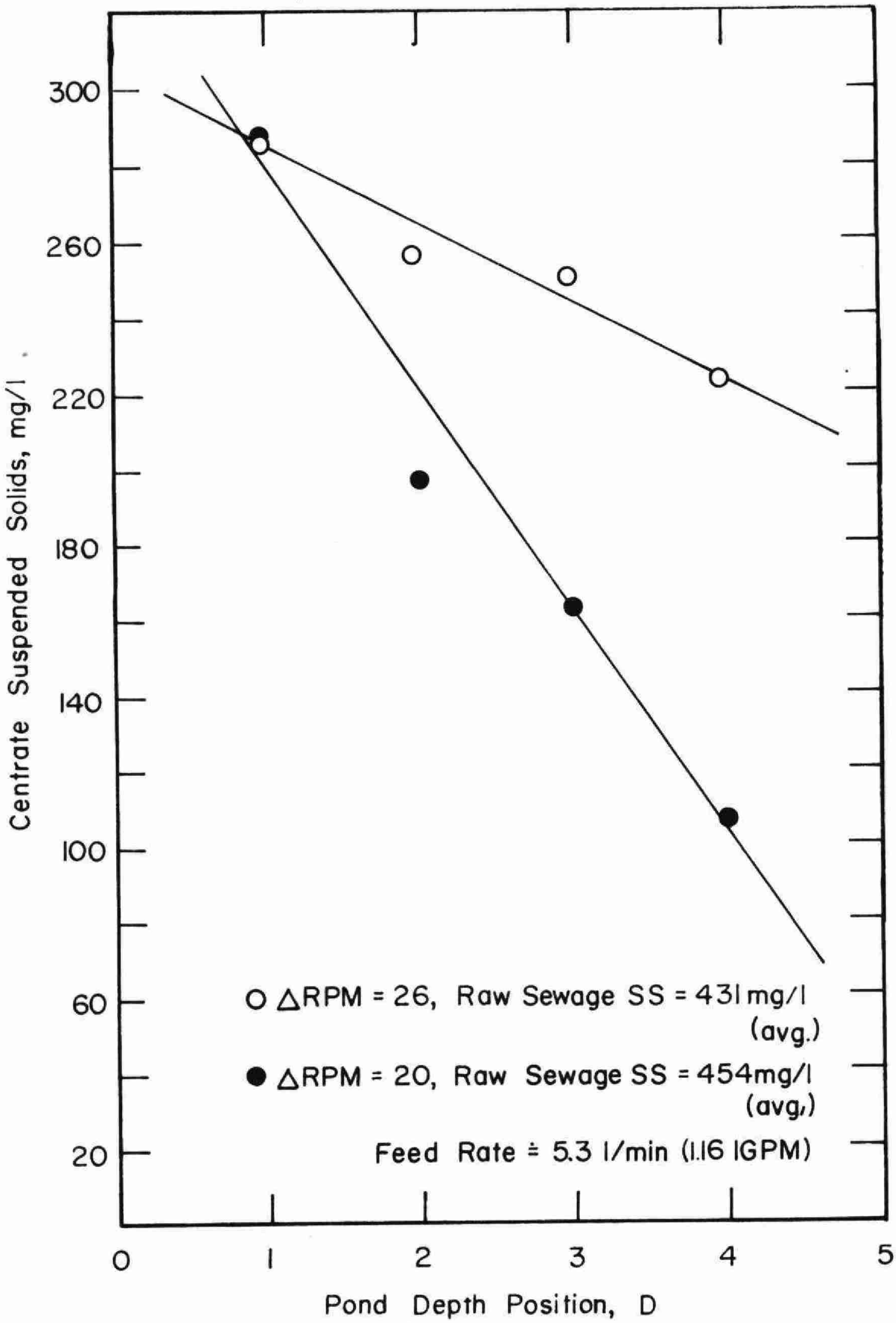


Figure 6. Effect of Pond Depth on Centrate Suspended Solids Concentration.

the solids residence time in the bowl. Thus from Figure 6 the lower the RPM differential and the greater the pond depth, the more solids will be removed from the liquid and thus the greater the clarifying ability of the centrifuge.

In general, reduction of centrate suspended solid may be attained by lowering the feed rate and the differential RPM, and increasing the pond depth. Optimal values are also dependent on the desired properties of the sludge cake as will be discussed in the next section.

4.2.3 Suspended Solids Removal Efficiencies and Cake Recovery

Because of the variation in sewage suspended solids, it is more meaningful to consider the solids removal efficiency (or solids recovery efficiency) as a performance parameter. The expression for calculating solids removal efficiency is given by equations (1) and (3) and the derivation of equation (1) is given in Appendix I.

Plots of the percent suspended solids removal and the percent cake solids vs the feed flowrate are plotted in Figures 7, 8 and 9. For the three sets of conditions analyzed, the solids removal efficiency and the cake solids composition exhibit decreasing values with increasing feedrate.

The removal efficiency decreases because of the increase in the solids composition of the centrate. The percent cake solids decreases because more solids are removed in the centrate over the same residence time.

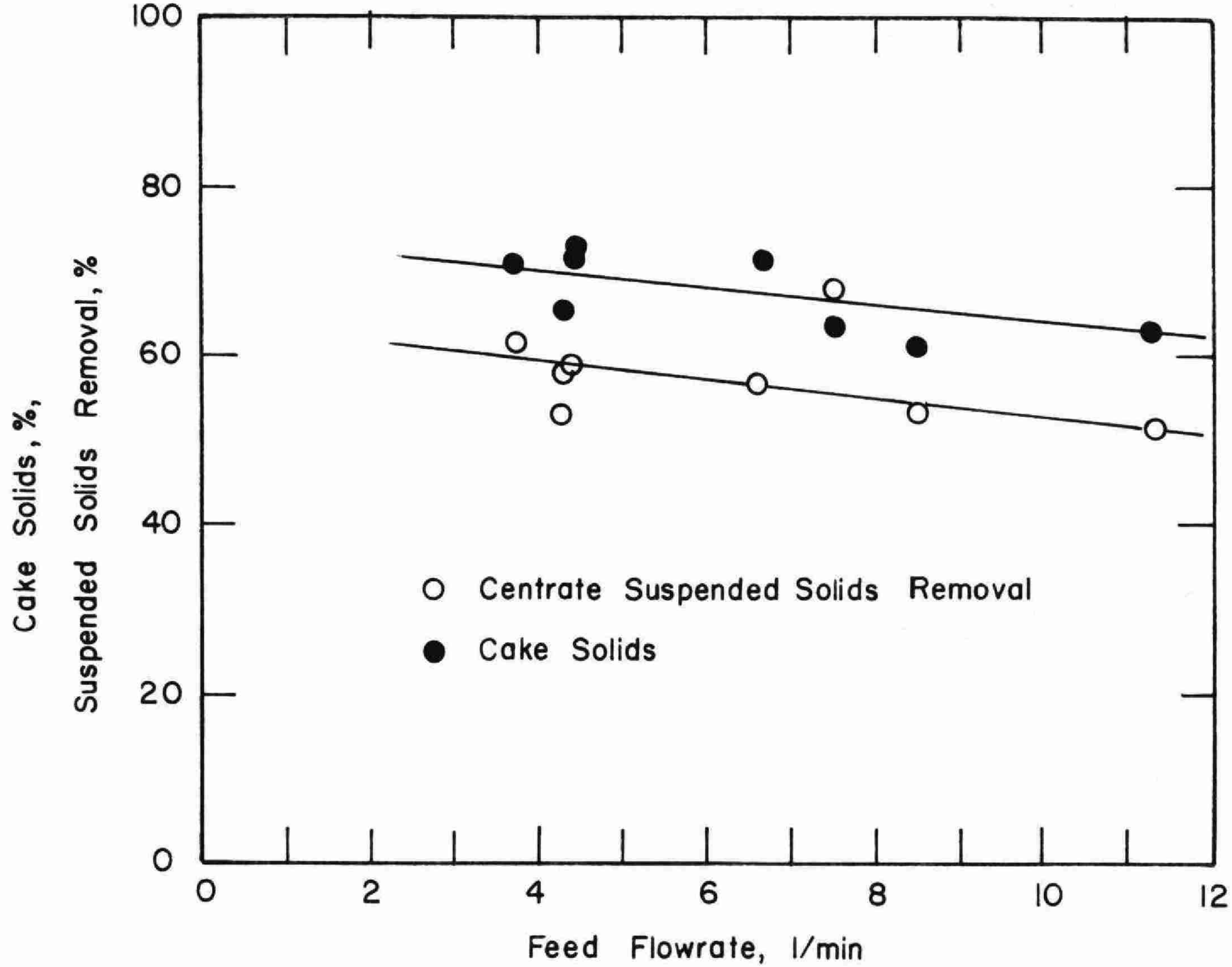


Figure 7. Effect of Feed Flowrates on Cake Solids and Suspended Solids Removal for RPM = 20 and D = 2.

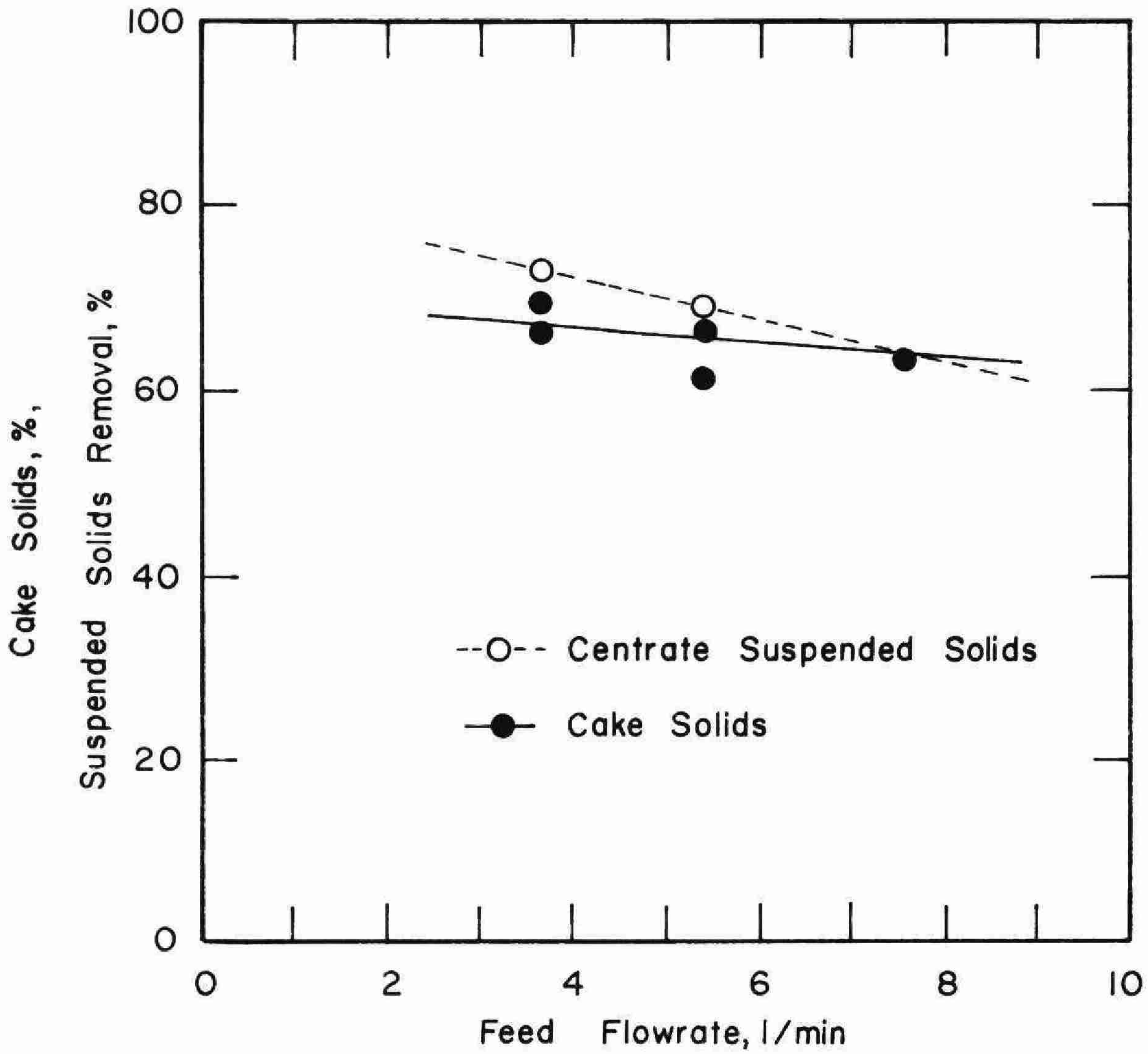


Figure 8. Centrate % SS Removal and Cake % Solids vs Feed Flowrate for ΔRPM = 20 and D = 4.

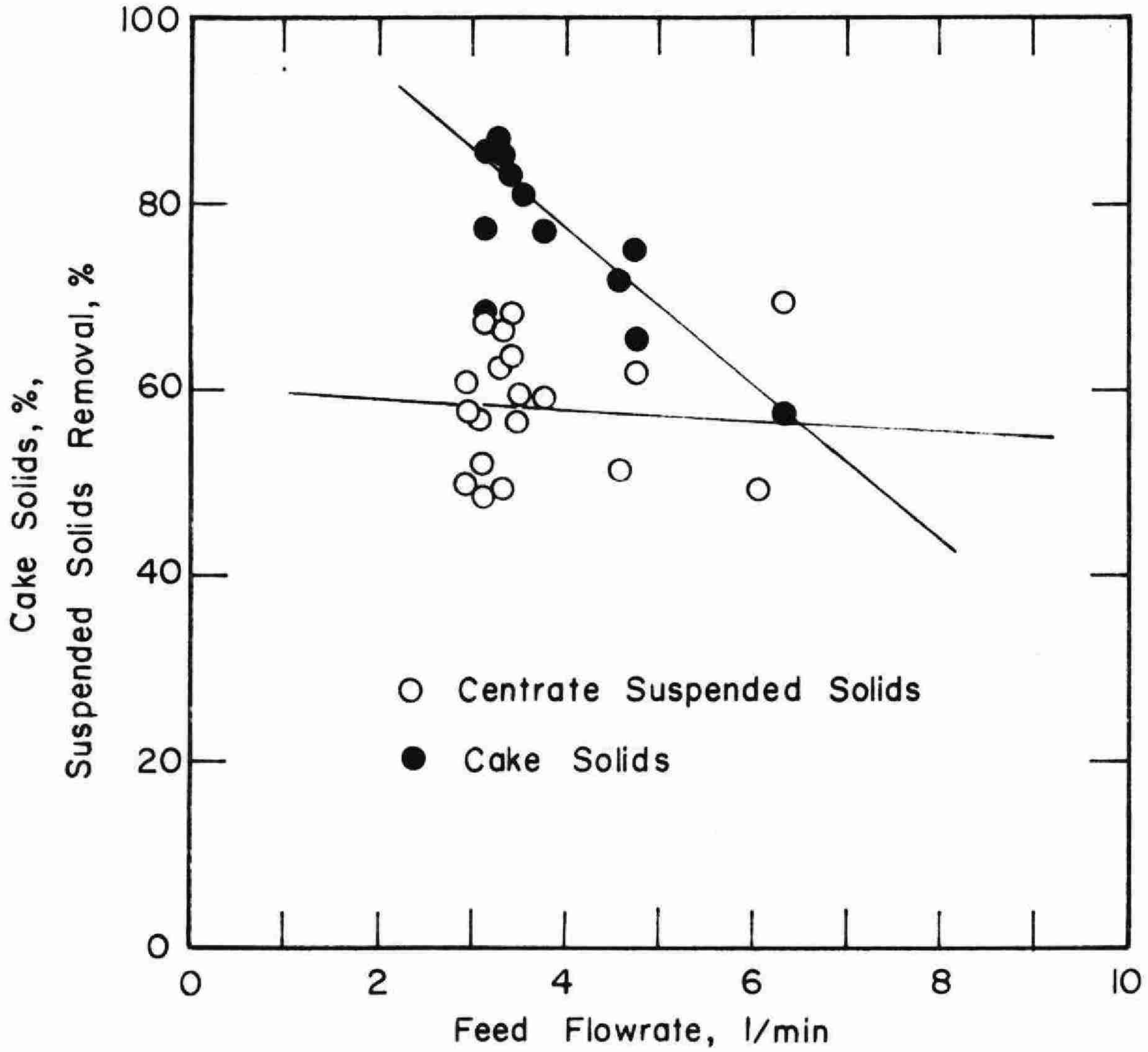


Figure 9. Centrate % SS Removal and Cake % Solids vs Feed Flowrate for ΔRPM = 12 and D = 4.

The calculated efficiency values are listed in Tables A1 and A2 in Appendix III.

Comparison of Figure 7 and 8, both at the same RPM differential, shows that an increase in pond depth from 2 to 4 increases the SS removal efficiency from a range of 52 to 60% to a range of 64 to 74%, that is, an average increase of about 12.3%. The cake composition decreased slightly over the range of feed rates considered. This again is a consequence of larger volume of water in the bowl, resulting from the increase in pond depth.

The effect of constant pond depths is illustrated in Figures 8 and 9, for ΔRPM's of 12 and 20. The increased residence time at the lower ΔRPM produced sludge cakes of higher percent solids than at the higher ΔRPM value. The cake solids then decreased rapidly with increasing feed rates, resulting in a drop of cake solids of 25% over the range of feed rates (from 3 to 6 l/min).

4.2.4 Pond Depth and RPM Differential

The variation of pond depth and RPM differential on removal efficiency and cake solids may be examined in greater detail for constant flowrate conditions using Figures 10, 11 and 12. Plots of η_{ss} vs D for values of ΔRPM of 20 and 26 in Figure 10 show a definite increase in η_{ss} with increasing D over the range of pond depth values examined. For the two ΔRPM's the lower ΔRPM value yielded the higher values of η_{ss}. The two differential RPM's resulted in η_{ss} values differing by approximately 20% over the range of pond depths at a feed rate of approximately 5.3 l/min (1.168 Igal/min).

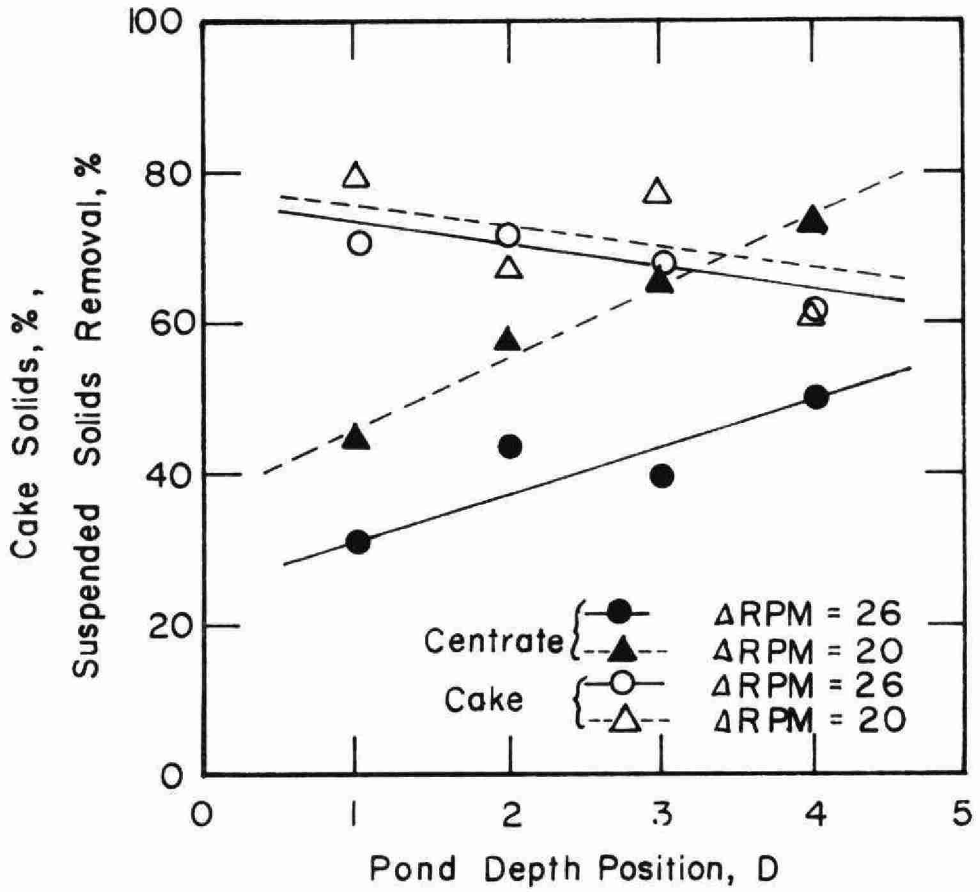


Figure 10. Effect of Pond Depth on Cake Solids and Suspended Solids Removal for a Feed Flowrate of 5.3 l/min (1.16 IGPM).

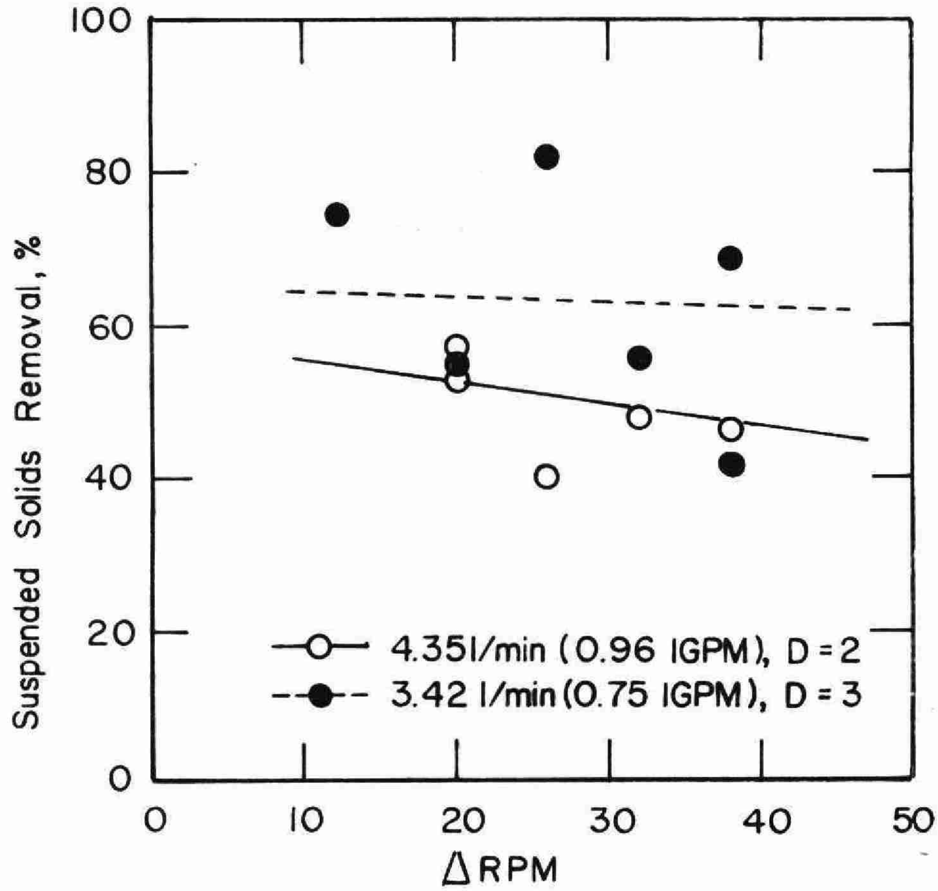


Figure 11. Variation of Suspended Solids Removal with ΔRPM.

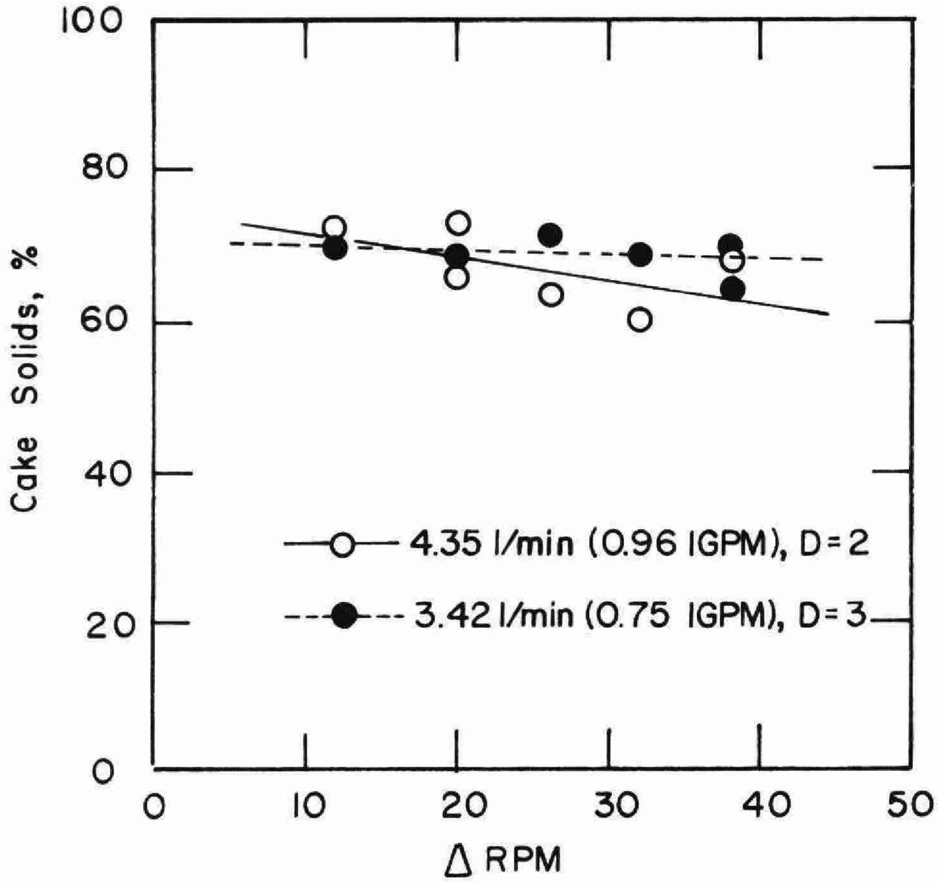


Figure 12. Variation of Cake Solids with ΔRPM.

On the same plot, the percent cake solids, C_S, show a decrease with increasing D for the two ΔRPM's. Thus, the increase in liquid in the bowl tends to produce a cake of higher moisture content which increases with increasing pond depth.

The variation in ΔRPM apparently has little effect on the value of C_S throughout the range of pond depths at the particular flow rate.

A closer examination of the effects of changes in ΔRPM may be made by plotting η_{ss} and C_S vs ΔRPM as in Figures 11 and 12. Again, these plots examine the behaviour at two constant values of flowrates. Although the data are more scattered for the pond depth values of 3, they are still significantly greater as predicted by Figures 7 and 8. At both pond depths there is a decrease with increasing ΔRPM.

From Figure 12, it is apparent that pond depth and angular velocity differential variations do not produce overly significant changes in the cake solids present at these flowrates.

4.3 Disc Centrifuge Evaluation

4.3.1 General Performance

The disc centrifuge produced centrates and sludges of a different nature as compared to that produced by the solid bowl centrifuges. The centrate from the disc machine appeared to have greater degree of clarity than the solid bowl centrate. The corresponding turbidity readings were also

lower, as monitored throughout the runs. The sludge cake was not of the same high solids consistency as produced by the solid bowl centrifuge, but was a dark watery liquid containing no more than 1.9% suspended solids.

4.3.2 Suspended Solids Removal

The values of suspended solids removal were calculated from equation (3) and plotted as in Figure 13 against feed flowrates, (the experimental data is tabulated in Table A2, Appendix III). All values, whether for full or partial desludge operation, were averaged for each cycle time.

The lines drawn through the points showed the 10 minute desludge cycle to yield higher removal efficiencies than the 20 minute cycle. Efficiencies for the 10 minute cycle ranged from 98% at 2.47 l/min to 50% at 19.0 l/min, while those for the 20 minute cycle varied from 78% at 3.8 l/min to 42% at 18.85 l/min.

The decrease in removal efficiency for each cycle time with increasing feed rates is due to the shorter detention time in the bowl, thus reducing the time for the centrifugal forces to effect a more complete sedimentation.

The effect of retention time may be illustrated more clearly if a dimensionless time is utilized. Defining a dimensionless time, t*, by the relation;

$$t^* = \frac{Q_f t}{V} \quad \text{..........} \quad (5)$$

where Q_f is the feed rate,

t is the cycle time, and

V is the volume of liquid retained by the centrifuge.

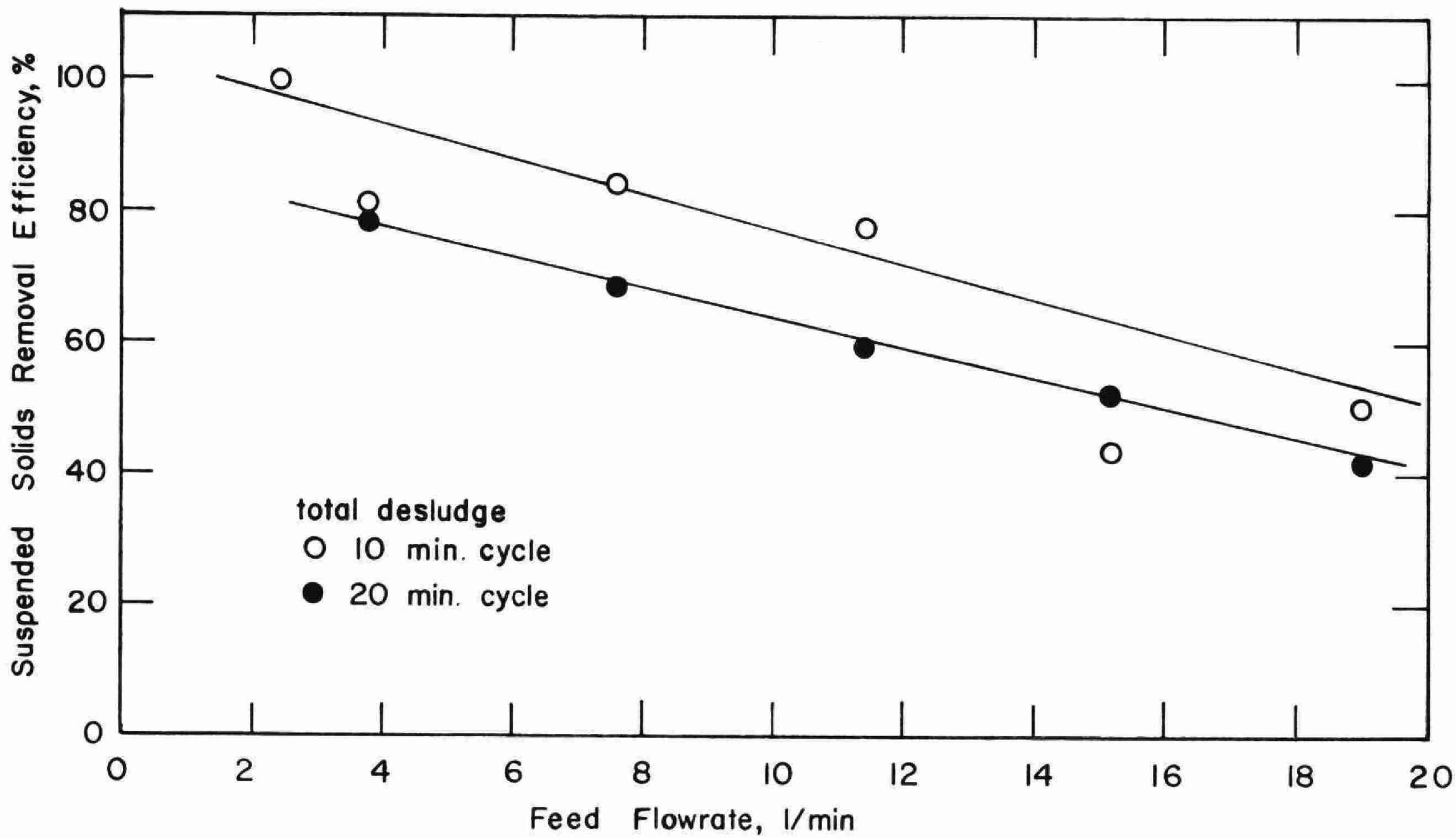


Figure 13. Effect of Feed Flowrate on Suspended Solids Removal for Disc Centrifuge.

The removal efficiency plotted on log-linear coordinates against t* yields a straight line relationship between the variables as shown on Figure 14. The variables obey the mathematical relationship,

$$\ln \eta_{ss} = \ln A - bt^* \qquad \ldots\ldots (6)$$

or

$$\eta_{ss} = A\exp(-bt^*) \qquad \ldots\ldots (7)$$

where A and b are constants and
in particular A = 100, b = 0.0145

The constants A and b are characteristic of the machine and the physical properties of the solid and liquid phases. Thus, increases in t* result in decreased SS removal efficiencies on an exponential basis.

4.3.3 Total and Partial Desludging

Figure 15 illustrates the effects of total and partial desludge methods on the solids removal efficiencies for 20 minute cycle times. The total desludge data are higher than the partial desludge values at the higher feed rates and lower at the lower feed rates.

At the higher flowrates the concentration of solids in the bowl is greater than at the lower values; therefore when the bowl is only partially emptied and then the remainder of the bowl volume filled with the feed, the overall concentration in the bowl is higher than that of the feed solution. Because of this, the suspended solids

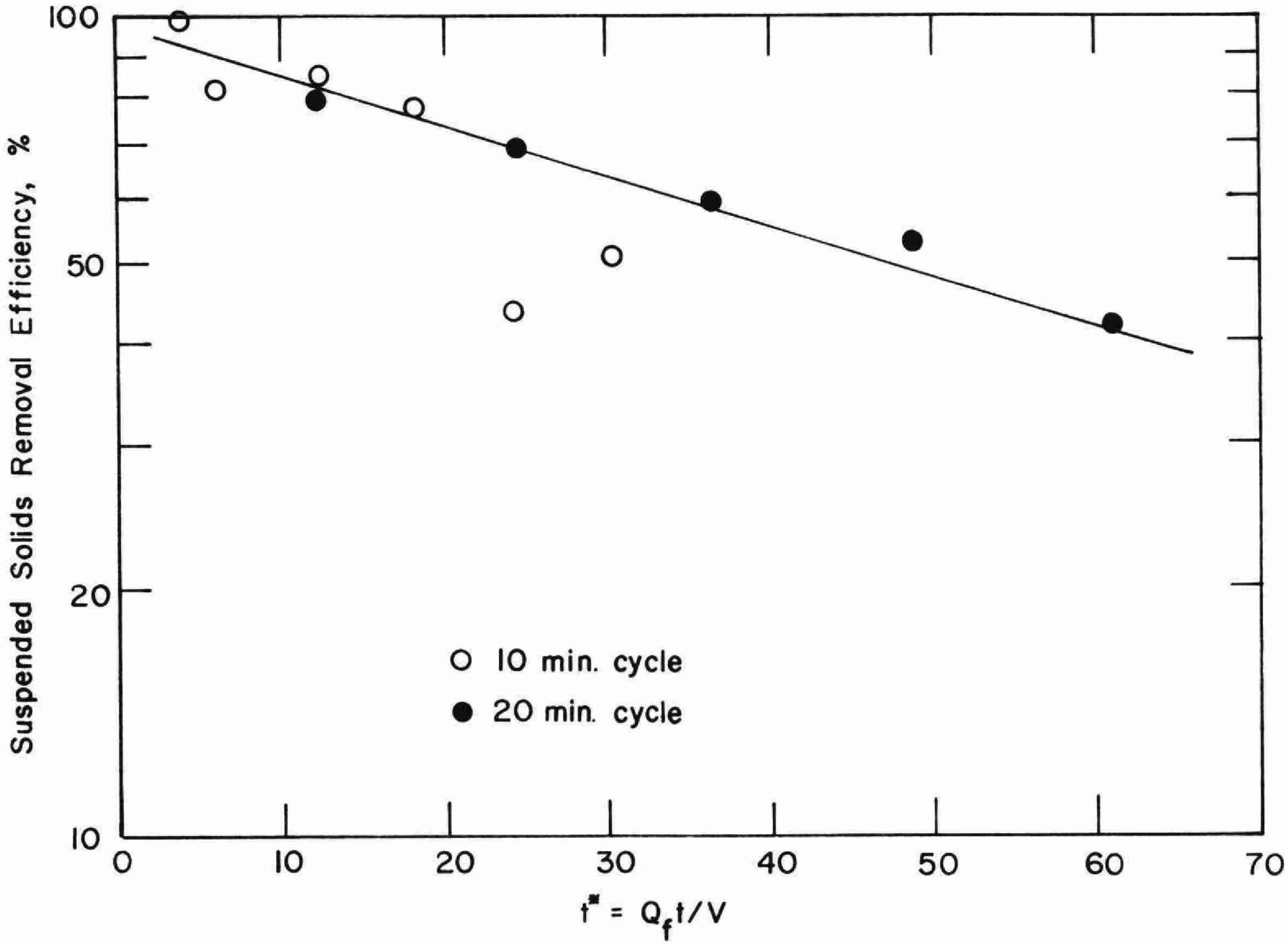


Figure 14. Graph of Suspended Solids Removal (Disc Centrifuge) Efficiency vs Dimensionless Variable, t*.

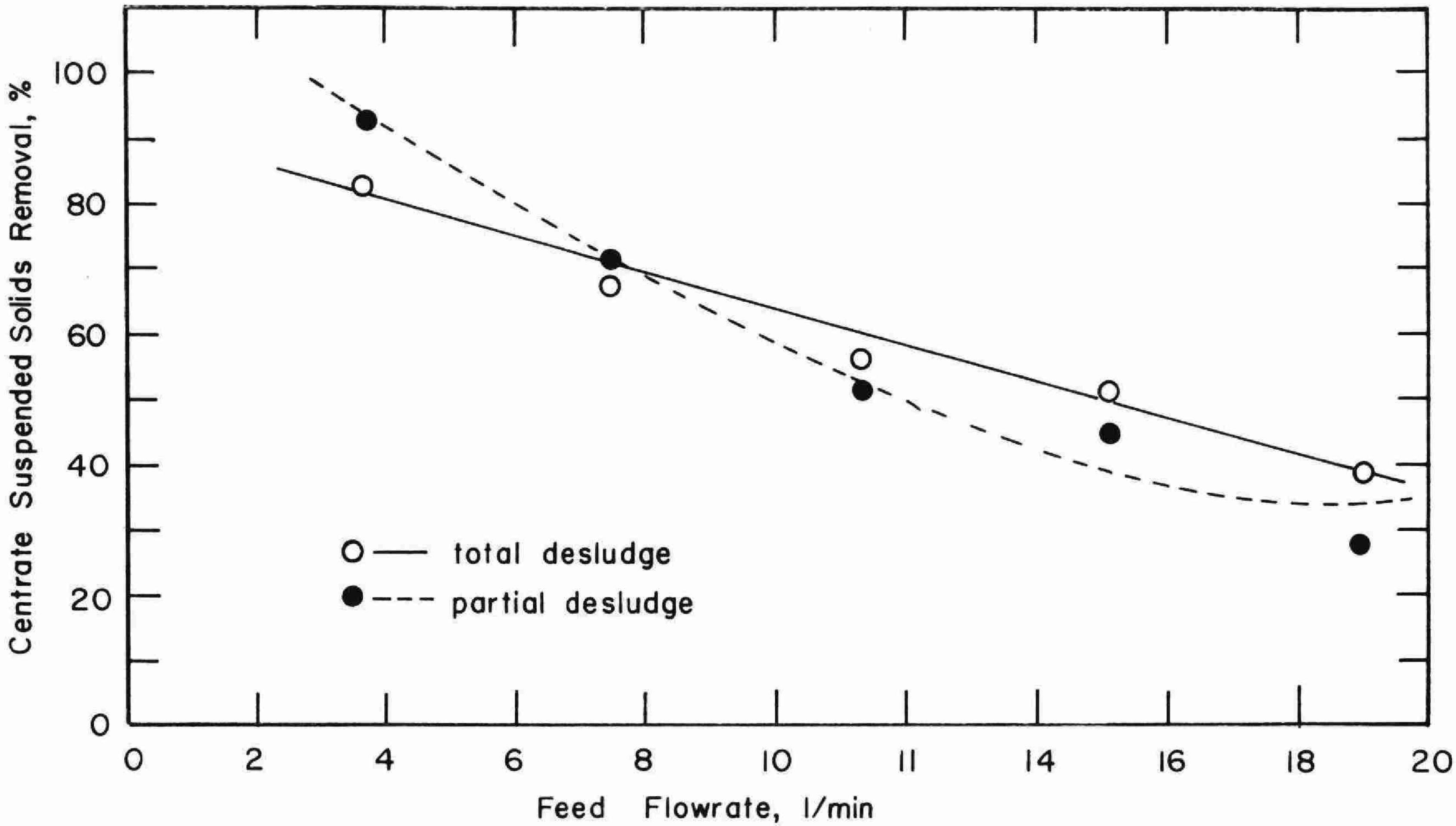


Figure 15. Effect of Feed Flowrate on Suspended Solids Removal for Total and Partial Desludge. (Disc Centrifuge)

concentration in the centrate increases. The removal efficiency being calculated on the basis of the feed concentration, will tend to be lower than for the complete desludge value. On the other hand, lower feed rates will produce a greater degree of clarification, so that after a partial desludge the liquid in the bowl will be of lower concentration than that of the feed. When the feed enters, the initial starting concentration at full bowl capacity will still be less than the feed concentration, thus the centrate SS will be lower than for the total desludge and the removal efficiencies will be higher.

4.3.4 Effect of Cycle Times

The effect of the length of the cycle times on the centrate composition and its variation with flowrate can be studied by plotting the centrate SS composition vs feed rate for the two cycle times studied, and for both total and partial desludges. From Figure 16, it is noted that the data splits into two types of curves. The 10 minute cycle values for both full and partial desludges are concave up, while the 20 minute values are concave down. In addition, the cycle length is indicative of the time for the sludge solids to build up within the bowl. As will be shown later, increasing the cycle time increases the sludge concentration in the bowl. The greater the quantity of sludge in the bowl, the larger the amount of carry-over of solids to the centrate. Therefore, for the same flow conditions, a longer cycle time will produce a greater concentration of suspended solids in the centrate. At the

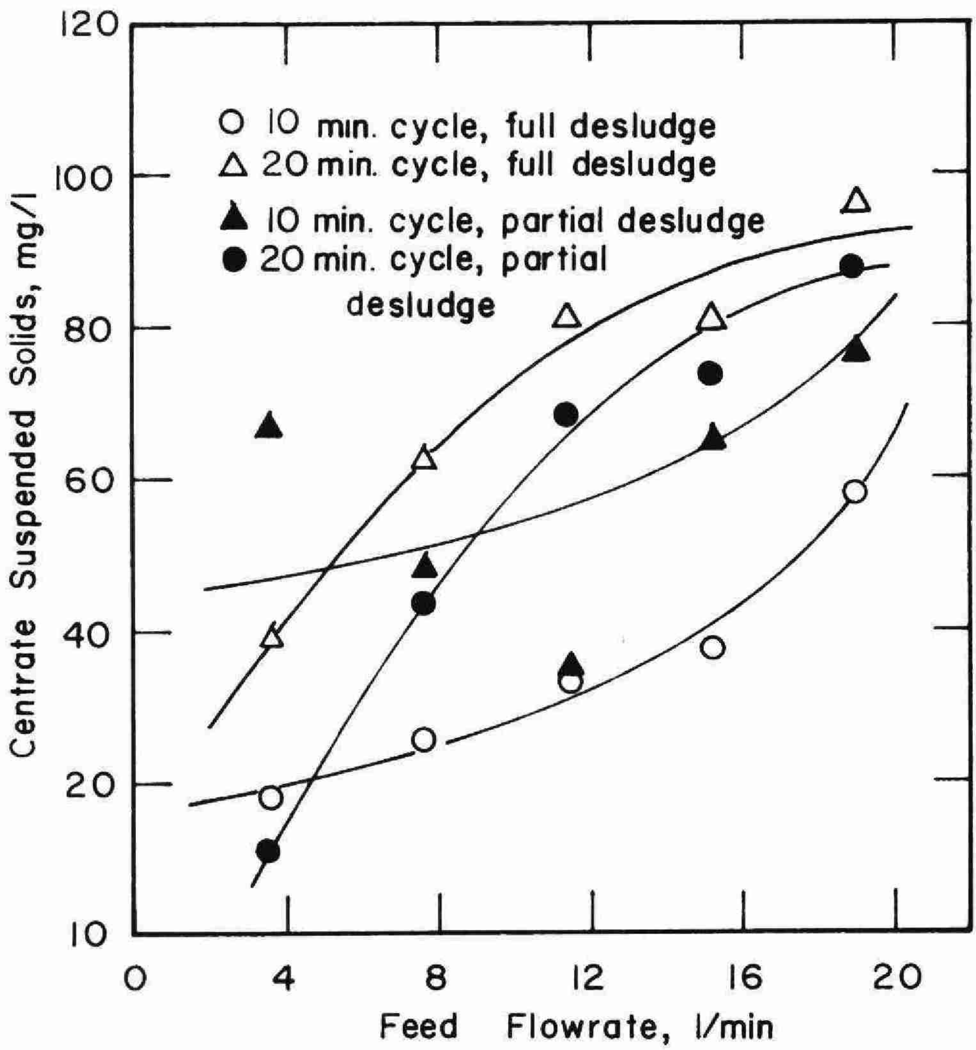


Figure 16. Variation of Centrate Suspended Solids Concentration with Feed Flowrate for Disc Centrifuge.

higher cycle time, the increase in solids carry-over occurs at a much greater rate with increasing feed rate than at the lower cycle times with increasing feed rate.

4.3.5 Sludge Concentrations

The variation of sludge concentration with feed rate is shown in Figure 17. In comparison with the results from the solid bowl centrifuge, the general trend of the sludge concentration is to increase with increasing feed rates rather than decrease as observed with the solid bowl machine. The reason for this opposite slope trend is the method of sludge or cake discharge. Whereas the solid bowl centrifuge discharged continuously, the disc centrifuge desludges the entire bowl or part of the bowl only after a fixed period of time; therefore, the quantity of suspended solids discharged from the bowl represents an accumulation which depends on cycle time and flowrate. Increasing flowrates bring more solids into the bowl per unit time resulting in a greater quantity of suspended solids being exposed to the centrifugal field.

From Figure 17, it appears that the concentration is increasing in a exponential manner with respect to feed flow. The effect of longer cycle times is readily apparent.

The lines for partial desludging have much greater slopes than those for total desludging. Also at the lower feed rates, the difference between 10 and 20 minute cycles is not as great as at the higher rates.

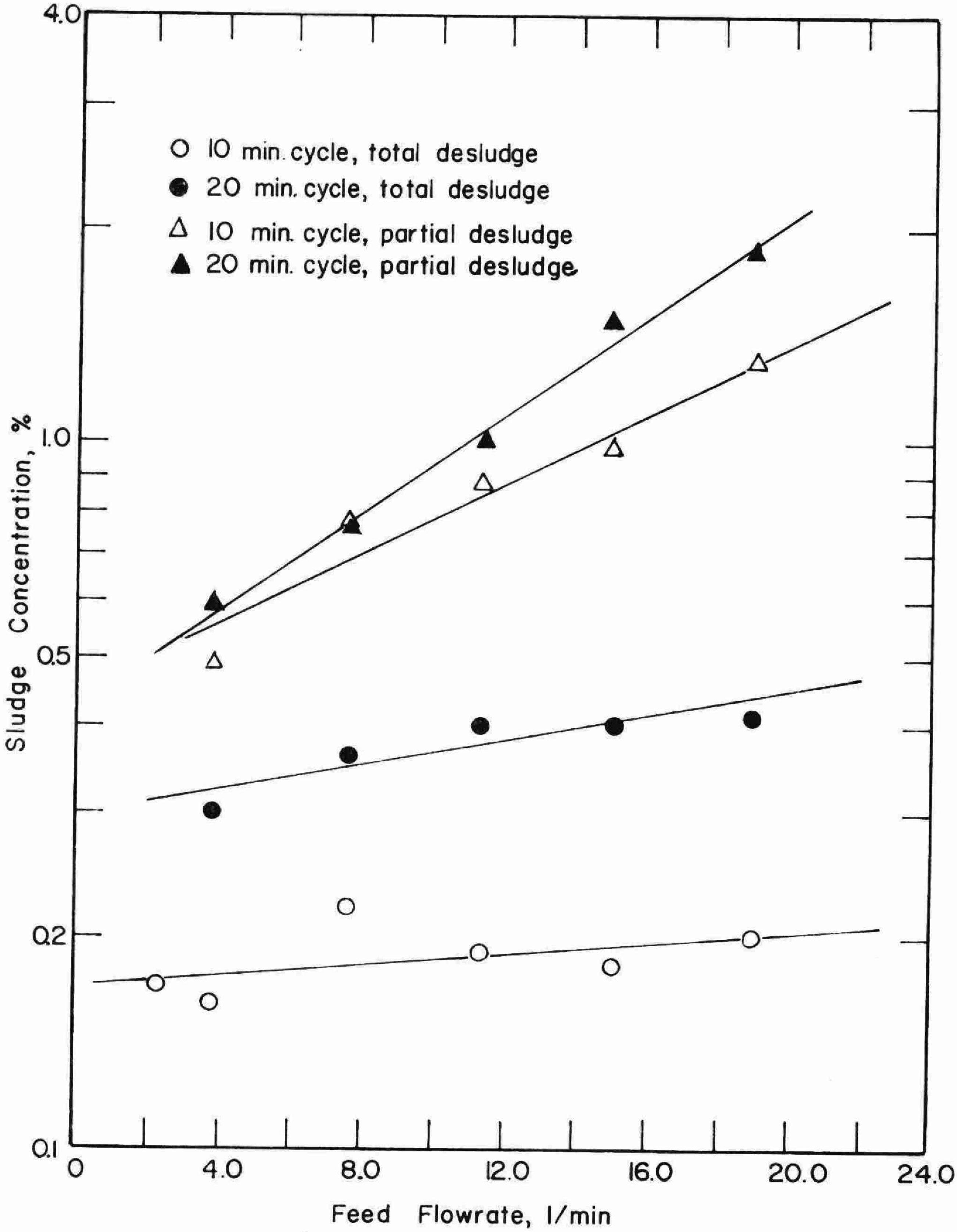


Figure 17. Variation of Sludge Concentration with Feed Flowrate for Disc Centrifuge.

4.3.6 Mathematical Model for Disc Centrifuge

The results of Figure 17 may be more readily understood if the system is approximated by a simple mathematical model.

Consider an unsteady state mass balance on the disc centrifuge for the duration of one cycle, from the time the bowl has been refilled to capacity after the previous discharge to the instant just before the end of the cycle when the sludge will discharge again.

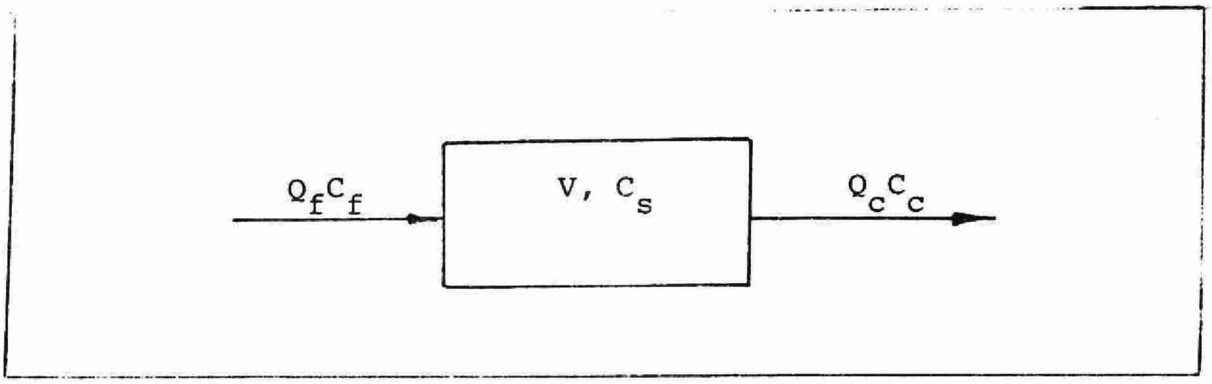
A mass balance based on Figure 18 is given by:

$$Q_f C_f - Q_c C_c(t) = \frac{d}{dt} (VC_s(t) \qquad ....(18)$$

where Q_f and Q_c are the flowrates of the feed and centrate, l/min.,
C_f is the feed concentration, mg/l
$C_c(t)$ is the centrate concentration as a function of time, mg/l.
$C_s(t)$ is the sludge concentration as a function of time, mg/l
t is time, min.
V is the bowl volume, l.

FIGURE 18

DISC CENTRIFUGE MASS BALANCE DIAGRAM



Over the cycle period, it will be assumed that the centrate and feed rates are identical, i.e.

$$Q_f = Q_c$$

and $dV/dt = 0$

Therefore, $\frac{Q_f}{V}\left[C_f - C_c(t)\right] = \frac{dc_s}{dt}$, and the

quantity, $\frac{V}{Q_f} = \tau$, where τ is a time constant, representing the liquid residence time in the bowl. The differential equation then becomes

$$\tau \frac{dC_{s(t)}}{dt} + C_c(t) = C_f \quad \ldots\ldots\ldots\ldots \quad (9)$$

In order to solve this equation, a relationship between $C_s(t)$ and $C_c(t)$ is required. A general representation of the relation between the variables may be approximated by a series solution;

$$C_c(t) = \sum_{i=o}^{\infty} a_i \, (C_s(t))^i, \quad \text{for all i such that}$$
$$i = 1,2 \quad .....,\infty$$
$$\text{and } a_i \text{ are constants.} \qquad ....(10)$$

Plotting sludge concentration against centrate concentration for the two cycle times and full desludge, a linear relation was obtained as in Figure 19. Therefore, the above series solution can be truncated to a first order approximation, namely,

$$C_c(t) = a_o + a_1 C_s(t) \qquad ....(11)$$

Substituting this linear relation into equation (9) the differential equation for the system becomes

$$\tau \frac{dC_s(t)}{dt} + a_1 \, C_s(t) = C_f - a_o \qquad ....(12)$$

which is classified as general first order linear differential equation. If the beginning of a cycle is taken to be time zero, with a bowl concentration of C_{so}, the boundary conditions for this differential equation are:

$$t = 0, \; C_s(0) = C_{so} \qquad ....(13)$$

In the case of a full desludge preceding the cycle, C_{so} will be equal to the feed concentration, C_f. For a partial desludge, the value of C_{so} may be given by

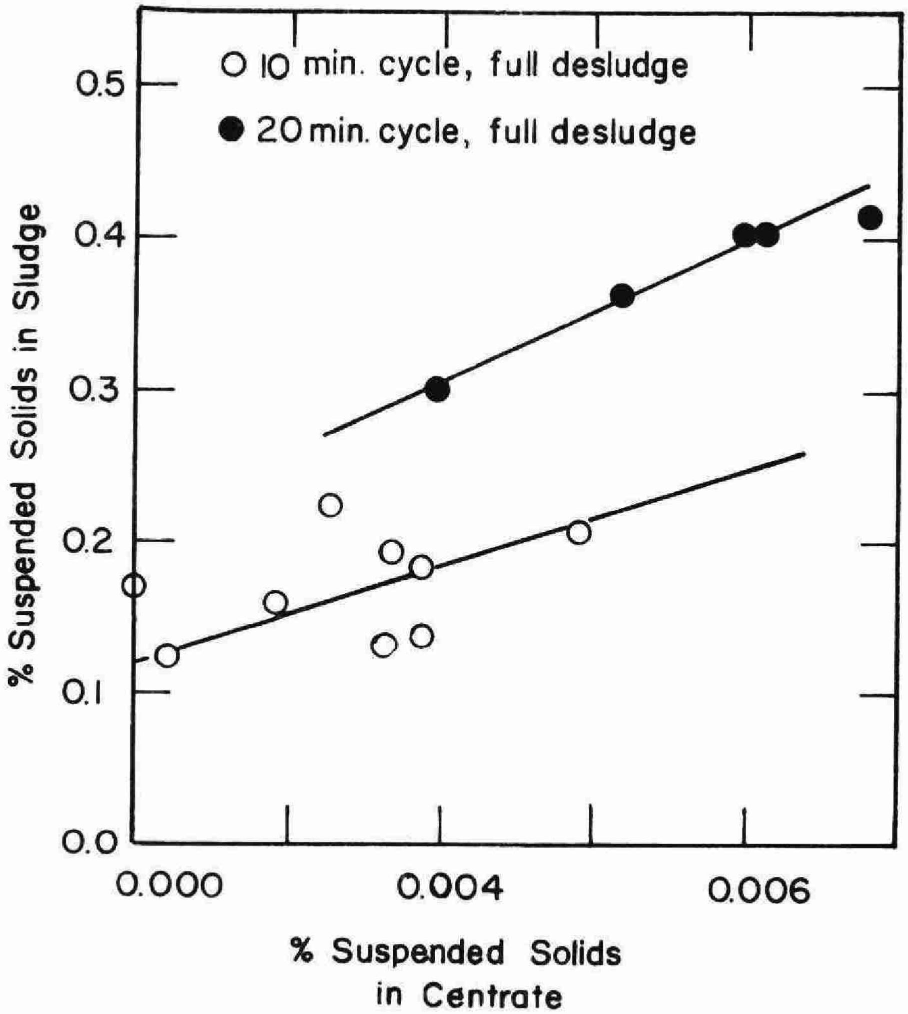


Figure 19. Concentration of Suspended Solids in Centrate and Sludge.

$$C_{so} = \frac{V_f C_f + V_b C_s(t_f)}{V} \qquad \text{..........(14)}$$

where V_b is the volume of sludge remaining in the bowl after partial discharge, l, V_f is the volume of feed to fill the bowl, l,

and $V = V_f + V_b$.

The solution of equation (12), has been worked out in detail in Appendix II. For the case of complete desludging the sludge concentration as a function of time and feed rate is given by the expression:

$$C_s(t) = C_f \exp\left(\frac{-a_1 Q_f t}{V}\right) + \left(\frac{C_f - a_o}{a_1}\right)\left[1 - \exp\left(\frac{-a\ Q_f t}{V}\right)\right] \quad \text{...(15)}$$

for the interval of one cycle. This equation shows that the sludge concentration varies exponentially with both Q_f and t. For behavior within a cycle, $C_s(t)$ varies only with t as Q_f is held constant. Thus the concentration will continually build up within the bowl as time increases so at a final time, t_f, at the end of a cycle, the desludging operation will give a sludge concentration of $C_s(t_f)$.

The variation of C_s at a fixed value of t_f with increasing feed rate explains the behaviour of the data of Figure 17.

It is also interesting to note that the argument of the exponential function, $-a_1Q_ft/V$, contains the time parameter, Q_ft/V utilized previously in correlating η_{ss} with flow rate and cycle time.

For conditions of a 20 minute cycle time with a full desludge and a feed concentration of 170 mg/l, values of $C_s(t_f)$ were calculated for different Q_f values and plotted as $C_s(t_f)$ against $1/Q_f$ in Figure 20. Also shown on the plot are the experimental values obtained for the same set of conditions. The general trend of the data is the same although the slopes are different. The deviation of the data arises from the accuracy of the values of C_f, ao and a_1 since the second term of equation (15) is extremely sensitive to changes in these parameters, and thus highly dependent on their accuracy. In addition, the value of C_f changed continually throughout the experiment as a result of the settling of the heavier particles during the run. In general, equation (15) depicts the variational trend of the sludge concentration with feed rate and feed concentration with an accuracy of better than $\pm$ 30%.

4.3.7 Variation of Sludge Concentration with Time

The variation of sludge concentration with time and the method of desludging is illustrated in Figure 21.

For the case of full desludging as shown by the solid line in the diagram, the bowl initially fills up over a period Δt_1 at concentration C_f. As the centrate

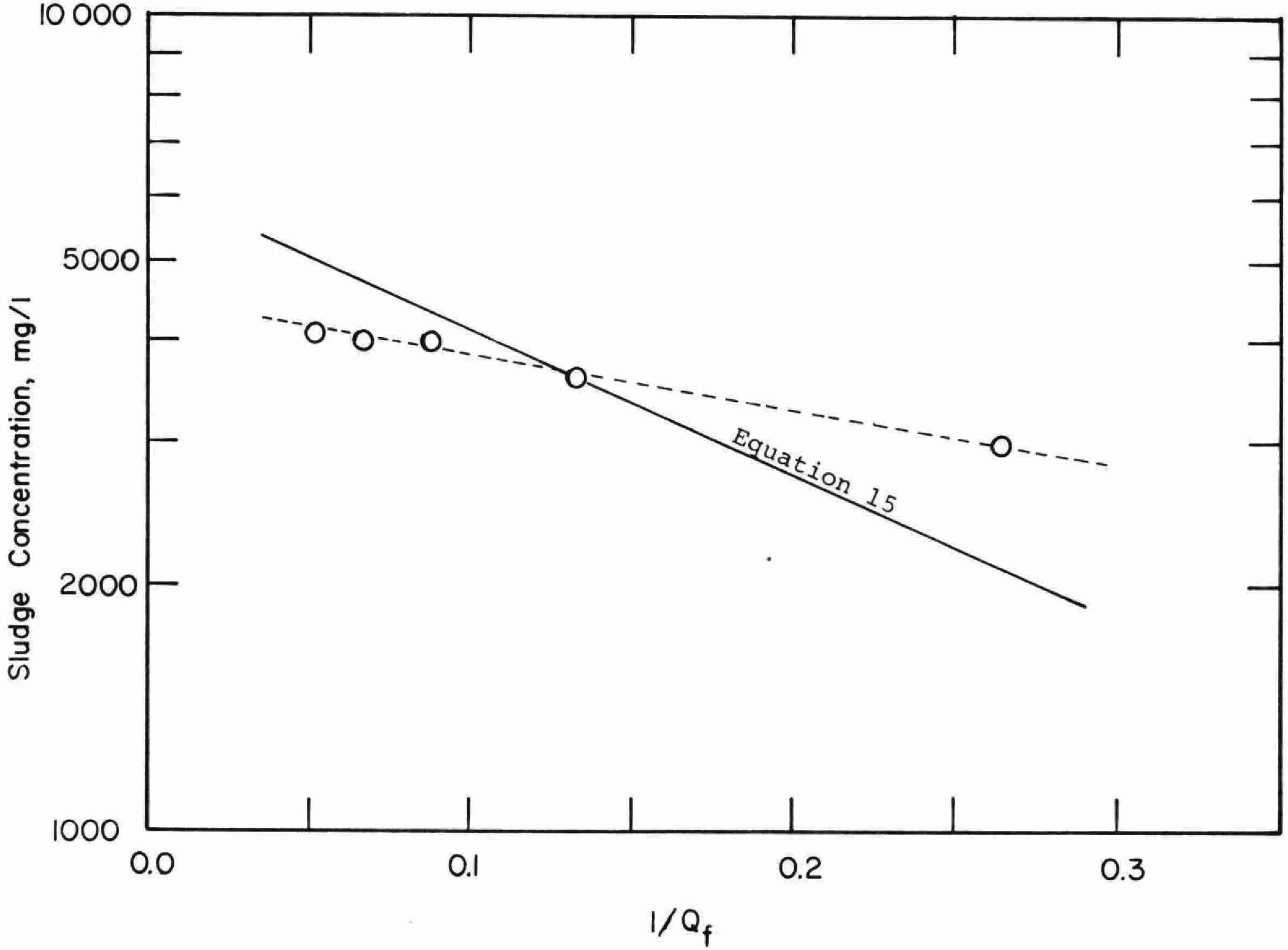


Figure 20. Sludge Concentration vs $1/Q_f$.

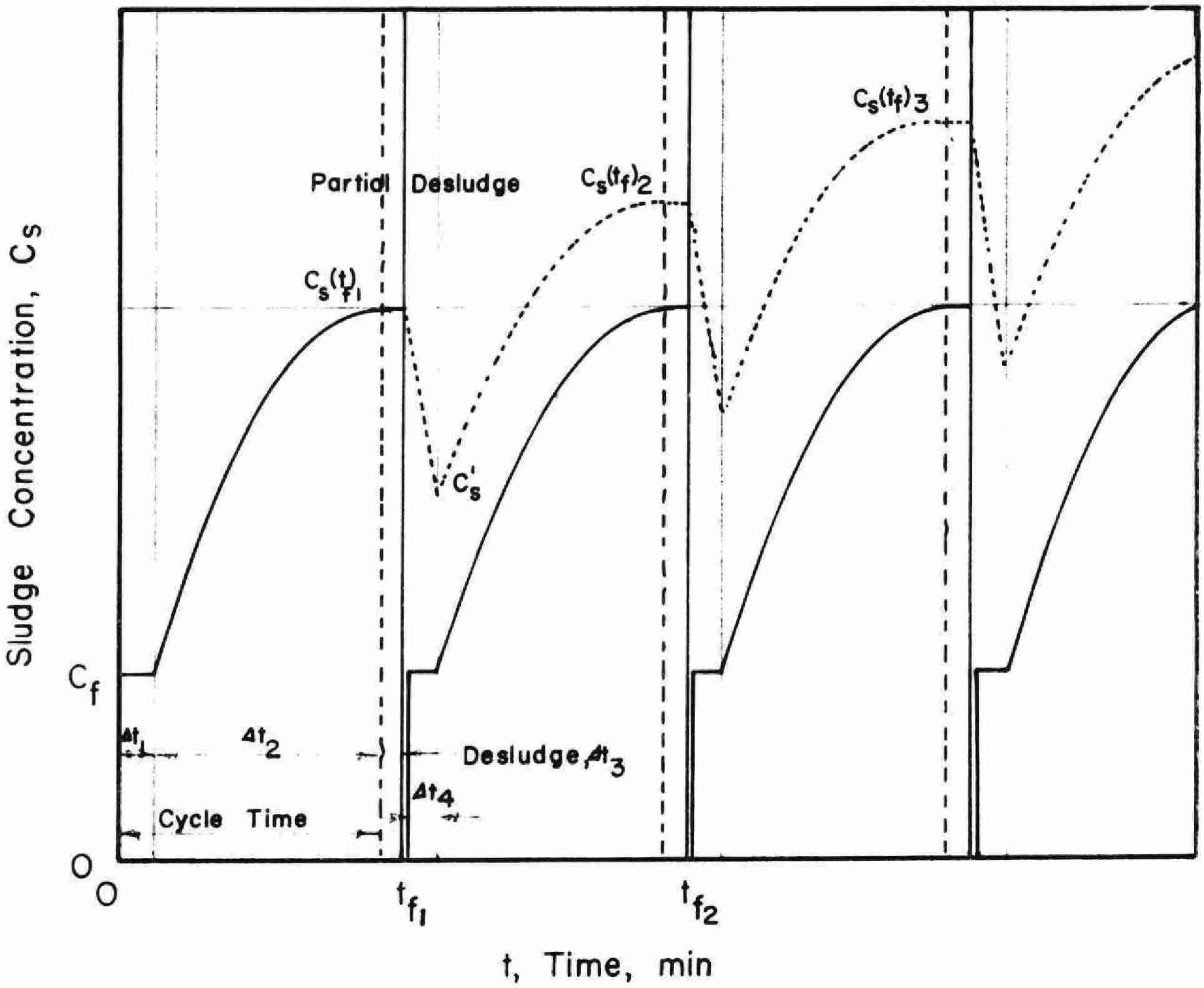


Figure 21. Full Desludge and Partial Desludge Cycles.

flow begins the accumulated concentration in the bowl increases according to equation (15) over the time, Δt_2, reaching the maximum value $C_s(t_f)$, at the end of the cycle time, t_f. As the bowl is desludged, this value of $C_s(t_f)$ remains constant over Δt_3 until the bowl is completely empty. After a time delay, Δt_4, the feed valve reopens and the cycle begins again.

For partial desludging the first cycle is identical to that of the full desludge case. Because the bowl does not empty completely, the value of C_s is constant over Δt_3 and Δt_4. As the bowl again begins to fill up, the value of C_s decreases due to dilution with feed until a value C'_s is reached when the bowl is completely full. As more feed is added the concentration increases exponentially to $C_s(t_f)_2$ at t_{f_2}. Thus the values of the sludge will be higher with partial desludging as compared to full desludging. After n number of cycles, a steady state value $C_s(t_f)_n$ should be reached. For this experiment samples were taken only at the end of the second cycle.

4.4 BOD and Phosphate Reduction

In addition to the analyses for suspended solids other variables were measured (Tables 3 & 4). No significant changes were noted for these data. The values for BOD and phosphorus did show some reduction due to the centrifugation process. These parameters and their removal efficiencies are shown in Table 4, for the solid bowl and disc centrifuges.

TABLE 3. AVERAGED SEWAGE AND CENTRATE CHARACTERISTICS

	pH	Phosphorus as P		Nitrogen as N				Formalin Turb. Units	% Turb.	Alkalinity	Conductivit µmhos/cm^3
		Total	Soluble	Free NH_3	Total Kjeld.	NO_2	NO_3				
A. SOLID BOWL CENTRIFUGE											
FEED	7.4	8.6	3.0	16	37	0.01	0.1	-	-	264	-
CENTRATE	7.5	6.4	3.0	16	36	0.01	<0.1	-	-	238	-
SLUDGE	-	-	-	-	-	-	-	-	-	-	-
B. DISC CENTRIFUGE											
FEED	7.5	8.2	5.1	24	40	0.02	<0.1	32	65	340	1390
CENTRATE	7.6	6.0	4.9	22.7	34	0.02	<0.1	16	20	328	1390
SLUDGE	6.5	174	100	72	450	0.05	0.65	>1000	>100	302	2170

TABLE 4. BOD AND PHOSPHATE REMOVAL

Run No.	Date 1973	Flow Rate (l/min)	ΔRPM (RPM)	D	BOD (mg/l) Raw Sewage	Centrate	Total Phosphorus (mg/l) Raw Sewage	Centrate	Removal Efficiency % BOD	TOT. P
A. SOLID BOWL CENTRIFUGE										
19	Aug.1	7.58	32	1	160	160	7.6	7.2	0	5.26
20	Aug.2	7.58	24	1	220	200	8.6	6.8	9.1	20.9
22	Aug.2	3.79	20	4	220	70	8.6	5.6	6.81	34.9
25	Aug.3	3.79	20	2	200	160	9.2	8.1	20.0	11.9
50	Aug.31	4.10	20	4	220	150	8.7	7.3	35.0	16.1
67	Sept.5	3.79	12	4	170	160	9.1	3.4	5.9	62.6
					AVERAGE EFFICIENCY				23%	25%
B. DISC CENTRIFUGE										
			Desludge Time							
10	Nov.8	3.79	10		130	70	10.0	7.5	46.2	25.0
16	Nov.8	18.95	20		220	80	14.0	12.0	18.2	16.3
22	Nov.14	3.79	20		180	110	13.0	9.5	38.9	27.0
31	Nov.14	18.95	20		190	140	9.8	8.8	26.3	10.8
33	Nov.16	3.79	20		140	36	5.4	4.1	74.3	24.1
37	Nov.16	3.79	20		65	19	5.7	3.8	70.8	33.3
42	Nov.16	3.79	20		80	42	6.0	4.0	47.5	33.3
46	Nov.19	3.79	20		40	19	4.1	3.1	52.5	24.4
47	Nov.20	3.79	20		130	55	7.0	4.6	57.7	34.3
50	Nov.20	3.79	20		100	38	6.4	4.1	62.0	36.0
51	Nov.21	3.79	20		130	60	7.2	4.7	53.8	34.6
52	Nov.21	3.79	20		110	45	7.0	4.5	59.1	35.7
53	Nov.22	3.79	30		140	100	10.0	7.4	28.6	26.0
57	Nov.22	3.79	30		140	70	9.4	6.2	50.0	34.1
					AVERAGE EFFICIENCY				49%	28%

The average efficiencies for BOD removal were 23% for the solid bowl centrifuge and 49% for the disc centrifuge.

The total phosphorus removal efficiencies were 25% and 28% for the solid bowl and disc respectively. These reductions in BOD and total phosphorus are due to the removal of solids containing BOD and phosphates. Thus, some BOD and phosphorus reduction can be expected in the raw sewage centrifugation process.

4.5 Filtration of Centrate

In order for centrifugation to be a viable unit operation for raw sewage treatment, it should be followed by some filtration process to remove most of the remaining suspended solids. Utilization of cartridge filters provided a simple means for investigating the effluent from a centrifuge-filter operation. The percentage of solids removal for several runs are listed in Table 5 along with the average values for the different sets of conditions. Overall, an average of approximately 80% removal can be expected with the filters used. The suspended solids remaining in the effluent fell within the range of 2 to 20 mg/l.

TABLE 5. CENTRATE SOLIDS REMOVAL BY FILTRATION

	Run Number	ΔRPM	D	% Removal	Average Solids Removal
Solid Bowl Centrifuge	51	20	4	100.0	
Flowrate ≃ 3.79 l/min.	52	20	4	93.77	92.3%
(1 USGM)	53	20	4	92.84	
	54	20	4	82.58	
	55	12	4	92.81	
	56	12	4	92.23	
	57	12	4	55.70	
	58	12	4	68.70	
	59	12	4	52.63	
	61	12	4	84.26	
	62	12	4	44.30	67.7%
	76	12	4	38.22	
	77	12	4	57.85	
	78	12	4	54.09	
	79	12	4	69.33	
	80	12	4	81.29	
	81	12	4	84.86	
	82	12	4	71.47	
		Cycle Time			
Disc Centrifuge	38	20 min.		79.0	
Flowrate ≃ 3.79 l/min	41	20 min.		100.0	
(1 USGM)	42	20 min.		100.0	80.8%
	43	20 min.		78.6	
	44	20 min.		46.4	
	48	30 min.		92.7	
	49	30 min.		42.3	
	51	30 min.		100.0	
	52	30 min.		78.4	74.5%
	53	30 min.		70.6	
	54	30 min.		70.2	
	55	30 min.		84.6	
	56	30 min.		68.6	
	57	30 min.		62.8	

5.0 SUMMARY AND COMPARISON

Ranges and variables and some average values for the two centrifuges are presented in Table 6. For further comparison, data from a primary sedimentation tank [8] are also included although the flow rate for the tank is several orders of magnitude greater.

The major performance parameter for comparison is the suspended solids removal efficiency. Overall, the centrifuge values are within the range of the sedimentation values but with the disc centrifuge giving the highest upper range values. These higher efficiencies occurred within the lower range of flowrates.

Most striking difference between the three systems is the sludge solids concentrations obtained from the solid bowl machine. In this case, the percent solids obtained had an average value of 70% and greatly exceeded the values for the disc machine and the primary sedimentation tank.

The reduction of BOD is a factor inherent in most suspended solid removal processes. The ranges of BOD removal efficiencies were similar for the three processes, and again the disc centrifuge upper range was higher than for the solid bowl or the primary sedimentation tank. Similarly, a higher phosphorus removal was also obtained for the disc machine.

Thus on a performance basis, the centrifuges are comparable to the sedimentation tank and it is assumed that this would be true at higher flow rates.

TABLE 6. SUMMARY AND COMPARISON OF RESULTS

	Solid Bowl Centrifuge	Disc Centri- fuge.	Primary Sediment- ation Tank[8] Circular **
Flow Rate	3 l/min to 11.37 l/min (0.678 to 2.58 IGPM)	2.47 l/min to 18.95 l/min (0.557 to 4.28 IGPM)	4370 l/min to 359,000 l/min (1.5 to 136.3 mgd.)
Suspended Solids			
Raw Sewage, mg/l	120 to 585	110 to 267*	187 to 354
Centrate mg/l	90 to 290	5 to 96	44 to 159
Removal Efficiency %	50 to 70	40 to 98	46 to 77
Sludge Solids, %	60 to 85	0.12 to 1.88	3.9 to 9.0
Average	70	-	-
BOD			
Raw Sewage, mg/l	160 to 220	65 to 220	90 to 383
Average mg/l	198	128	-
Centrate mg/l	70 to 160	19 to 140	43 to 291
Average	150	63	-
Removal Efficiency %	5.9 to 68.1	18 to 74	22.5 to 52
Average %	23	49	-
Total Phosphate			
Raw Sewage, mg/l	7.6 to 9.2	4.1 to 14	-
Average %	8.6	8.2	-
Centrate mg/l	3.4 to 8.1	3.1 to 12	-
Average %	6.4	6.0	-
Removal Efficiency %	5.26 to 62.6	16 to 36	-
Average %	25	28	-
Detention Time	-	0.32 min to 2.43 min.	1.6 hrs to 4.32 hrs

* Screened Raw Sewage.

** Range of values for 15 treatment systems operating on domestic and industrial sewage.

6.0 OTHER SLUDGE THICKENING METHODS

Sludge from primary treatment processes require dewatering before primary disposal. In addition to centrifugation, methods commonly used today are air drying, vacuum filtration and machanical separation. Of these processes, sand beds are most common, but because of increased cost of land and labour mechanical dewatering devices are now more widely used. Since 1960, there has been an upsurge in vacuum filtration because of improved filter media, higher costs for competing methods and the growing popularity of sludge incineration. The centrifuge is seeing wider use because of the design improvements in the solid bowl, moderate operating costs and low space requirement. The major problem with centrifugation is that the centrate contains fine solids and if recycled, they can build up in the system. The problem is greater with biological sludges from secondary treatment systems.

Chemical flocculants can be used to improve centrate qualities [3] but their use can increase the operating costs of the process significantly.

Since vacuum filtration remains the predominant mechanical dewatering device, a comparison between vacuum filtration and centrifugation is drawn in Table 7. The media commonly used in vacuum filtration are cloth, steel mesh and tightly wound coil springs.

Table 7 indicates lower moisture contents in the sludge cake were obtained by centrifugation and for lower feed concentrations than the two filters. Also coagulants were used in pretreatment in each of the filtration cases.

TABLE 7. COMPARISON BETWEEN FILTRATION AND CENTRIFUGATION

	Feed	Feed Concentration	Coagulants	Cake Moisture
Vacuum Filter [9]	Primary Sludge	16% solids	9.9% CaO	54%
Coilspring Filter [9]	Primary Sludge	6.6% solids	8.3% CaO + 3.0% $FeCl_3$	73%
Solid Bowl Centrifuge (this study)	Raw Sewage	0.012% to 0.05% solids	None	40 to 15% Average = 30%

7.0 CONCLUSIONS

Centrifugation of raw sewage has been investigated using a solid bowl centrifuge and a disc centrifuge. The major variables considered were the suspended solids removal efficiency and the concentration of solids in the sludge. The operating parameters examined were flowrate, pool depth, RPM differential for the solid bowl unit, and cycle times and desludging methods for the disc unit. The results obtained were explained in terms of the effects of the above mentioned parameters on the solids residence times in the bowls.

The major conclusions that can be obtained from this study are:

1. Both centrifuges can be operated on a continuous basis regardless of input concentration of suspended solids, although preliminary screening of sewage prior to entry to the disc unit is necessary to avoid plugging the discs.
2. Higher quality effluents and, consequently, greater removal efficiencies were obtained using the disc unit. Removal values as high as 98% were obtained.
3. The sludge produced by the solid bowl unit contained an average of 70% solids and was of a form that was easily handled for further disposal. The disc unit produced a sludge which never contained more than 1.9% solids. The solids present in the disc sludge were extremely fine and could pose a problem with respect to further disposal.

4. The suspended solids removal efficiency for the solid bowl unit was observed to increase with increasing pond depths and decrease with increasing flow-rates, and angular velocity differentials. The disc unit displayed higher efficiencies for lower flowrates, shorter cycle times, and full desludging.
5. The sludge cake quality from the solid bowl unit decreased with increasing flowrates, pond depths and RPM differentials. The concentration of sludge solids from the disc unit increased with increasing flowrates, cycle times and the use of the partial desludging operation.
6. Suspended solids removal efficiencies for the disc unit can be correlated using the dimensionless variable, $t^* = Q_f t/V$, involving both flowrate and cycle time. The general expression for this correlation is:

$$\eta_{ss} = Ae^{-bt^*}$$

7. The mathematical model derived for analyzing the variation in sludge concentrations from the disc centrifuge predicts concentration values with an error of better than $\pm$ 30% for the particular centrifuge and flow conditions under investigation.

8. Reductions of BOD and phosphate were obtained with both centrifuges as a result of the removal of BOD and phosphate containing solids. The average values of BOD removal efficiencies were 23% and 49% for the solid bowl and disc units respectively. Phosphate removal efficiencies averaged 25% and 28% for the solid bowl and disc units respectively.
9. From a general comparison with vacuum filtration, the solid bowl centrifuge sludge cake had lower moisture content than the cakes from filters and did not require coagulants.
10. In general, the solid bowl centrifuge has several definite advantages over primary gravity sedimentation with respect to space requirements, odour control, residence time, effluent quality, sludge cake quality, and ease of cake handling. It is thus readily adaptable for use in an advanced physical treatment system.
11. The major disadvantage of centrifugation as applied to raw sewage treatment is the high energy requirement necessary for its operation, especially when compared to primary sedimentation which is considered to be a low energy process.

NOMENCLATURE

a_i	constant, for i = 1,2 ......
A	constant
b	constant
C	concentration, mg/l
e	constant, base of $\log_e$ or ln
i	index, from 1 to ∞
Q	flowrate, l/min
g	gravitational acceleration = 980 cm/sec^2 (32.2 ft/sec^2)
r	distance of a particle from axis of rotation, cm (ft)
t	time, min
t*	defined by equation (5), dimensionless
t_f	cycle time, min
V	Volume of bowl, l
V_b	Volume of sludge remaining in bowl, l
V_f	Volume of feed to fill bowl, l.

SUPERCRIPTS

C	centrate
R	raw sewage

SUBSCRIPTS

C	centrate
DS	dissolved solids
f	feed
S	sludge
SS	suspended solids
TS	total solids

GREEK LETTERS

η_{ss}	efficiency of suspended solids removal
τ	time constant, min
Σ	performance factor, m^2 (ft^2)
$\sum_{i=1}^{\infty}$	summation symbol
ω	angular velocity, rad/sec (RPM)

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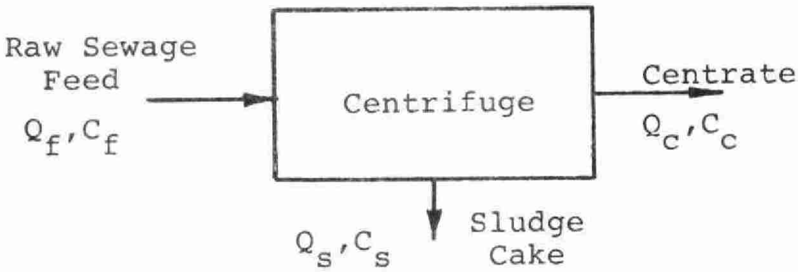
APPENDIX I

DERIVATION OF SOLIDS REMOVAL EFFICIENCY FOR SOLID BOWL CENTRIFUGE

Consider a mass balance on the solid bowl Centrifuge operating under steady state conditions.



Solids input rate = $Q_f C_f$

Solids output rate = $Q_c Q_c + Q_s C_s$

$$\therefore \quad Q_f C_f = Q_c C_c + Q_s C_s$$

and $Q_f = Q_s + Q_c$

or $Q_s = Q_f - Q_c$

$$\therefore \quad Q_f C_f = Q_c C_c + (Q_f - Q_c)\, C_s$$

ie: $Q_f \,(C_f - C_f) = Q_c (C_s - C_c)$

$$\therefore \quad Q_f = Q_c \left(\frac{C_s - C_c}{C_s - C_f} \right) \qquad \ldots\ldots \text{(A1)}$$

$$\text{Ratio of solid removal} = \eta = \left(\frac{Q_s C_s}{Q_s C_f} \right)$$

$$= \left(\frac{Q_f - Q_c}{Q_f} \right) \frac{C_s}{C_f} = \left(1 - \frac{Q_c}{f} \right) \frac{C_s}{C_f}$$

Substituting

$$\eta = \left[1 - \left(\frac{C_s - C_f}{C_s - C_c}\right)\right]\frac{C_s}{C_f} \quad \ldots\ldots \quad (A2)$$

$$\therefore \quad \eta = \left(\frac{C_f - C_c}{C_s - C_c}\right) \frac{C_s}{C_f} \quad \ldots\ldots \quad (A3)$$

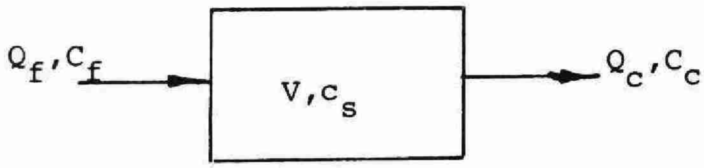
which is the desired expression for solids removal efficiency.

Because the cake concentration is much greater than the centrate concentration, ie: $C_s >> C_c$, equation (A3) reduces to:

$$\eta = \frac{C_f - C_c}{C_f} \quad \ldots\ldots(3)$$

APPENDIX II

MODEL FOR DISC CENTRIFUGE



$$Q_f C_f - Q_c C_c(t) = \frac{d(VC_s)}{dt}$$

Assume $Q_f = Q_c$ and $\frac{dV}{dt} = 0$

$$\therefore \frac{Q_c}{V} (C_f - C_c(t)) = \frac{dc_s}{dt}$$

$\frac{V}{Q_c} = \tau =$ time constant for system

$$\therefore \tau \frac{dC_s(t)}{dt} + C_c(t) = C_f$$

or $$\frac{dC_s(t)}{dt} + \frac{C_c(t)}{\tau} = \frac{C_f}{\tau}$$

Assume there is a linear relationship between C_s and C_c

ie: $C_c = a + bC_s$

$$\therefore \frac{dC_s(t)}{dt} + (a + bC_s(t))/\tau = C_f/\tau$$

or $$\frac{dC_s(t)}{dt} + \frac{b}{\tau} C_s(t) = \frac{C_f}{\tau} - \frac{a}{\tau}$$

Let $K_1 = \frac{b}{\tau}$

and $K_2 = \frac{C_f}{\tau} - \frac{a}{\tau}$

$$\therefore \quad \frac{dC_s(t)}{dt} + K_1 C_s(t) = K_2$$

B.C. $t = 0$, $C_s(t) = C_{so}$

$$\frac{d}{dt} (C_s e^{K_1 t}) = e^{K_1 t} K_2$$

$$\text{or } C_s e^{K_1 t} = \int e^{K_1 t} K_2 dt + C'$$

$$\text{ie: } C_s = e^{-K_1 t} K_2 \int e^{K_1 t} dt + C' e^{-K_1 t}$$

$$\text{or } C_s = \frac{K_2}{K_1} + C' e^{-K_1 t}$$

for $C_s(0) = C_{so}$

$$C_{so} = \frac{K_2}{K_1} + C' \text{ or } C' = C_{so} - \frac{K_2}{K_1}$$

$$\therefore \quad C_s = \frac{K_2}{K_1} + C_{so} e^{-K_1 t} - \frac{K_2}{K_1} e^{-K_1 t}$$

$$\text{or } C_s = C_{so} e^{-bt/\tau} + \left(\frac{C_f - a}{b}\right) (1 - e^{-bt/\tau})$$

if we assume $C_{so} = C_f$

$$\therefore C_s = C_f \exp \left(\frac{-bQ_c t}{V}\right) + \left(\frac{C_f - a}{b}\right)\left[1 - \exp \frac{-bQ_c t}{V}\right]$$

for the interval of one cycle.

APPENDIX III

EXPERIMENTAL RESULTS

TABLE A1

RESULTS SUPER-D CANTER

SOLID BOWL CENTRIFUGE

Run.	Date. 1973	Q(USGM)	ΔRPM	D	Solids Removal Efficiency, %	% Cake Solids
6	Jul.25	1.4	20	3	62.1	66.2
7	" 25	1.2	20	3	66.0	68.2
8	" 26	0.9	38	3	68.2	64.0
9	" 26	0.9	26	3	82.0	71.4
10	" 26	0.9	12	3	74.4	69.5
11	" 26	0.9	20	3	54.4	68.2
12	" 26	0.9	32	3	55.3	68.8
13	" 26	0.85	38	3	42.0	69.6
14	Jul.31	2.0	32	3	49.7	61.3
15	" 31	2.0	20	3	65.0	65.2
16	" 31	1.5	20	3	D> C	70.0
17	" 31	1.5	32	3	D> C	65.8
18	" 31	1.0	32	3	D> C	68.5
19	Aug. 1	2.0	20	1	41.0	71.0
20	" 2	2.0	20	1	67.5	72.9
21	" 2	1.0	20	1	D> C	75.5
22	" 2	1.0	20	4	D> C	69.0
23	" 2	2.0	20	4	D> C	63.4
24	" 3	2.0	20	2	67.5	63.5
25	" 3	1.0	20	2	61.5	70.6
26	" 15	3.0	20	2	51.5	62.2
27	" 15	2.25	20	2	53.0	60.9
28	" 15	1.75	20	2	56.8	70.9
29	" 15	1.15	20	2	57.6	65.3
30	" 15	.65	20	2	58.9	71.2
31	" 24	1.15	38	2	46.7	67.8
32	" 24	1.15	32	2	48.0	60.0
33	" 24	1.15	26	2	40.2	63.5
34	" 24	1.15	20	2	52.5	72.9
35	" 24	1.15	12	2	-27.2 (C >R)	72.6
36	" 28	1.42	26	4	49.9	62.0
37	" 28	1.42	26	3	39.4	67.2
38	" 28	1.42	26	2	43.7	71.6
39	" 28	1.42	26	1	30.9	70.5
40	" 29	1.37	20	1	44.5	79.5
41	" 29	1.42	20	2	57.3	67.5
42	" 29	1.39	20	3	66.5	77.4
43	" 29	1.42	20	4	73.0	61.3
44	" 30	1.37	38	4	52.7	66.3

TABLE A1 (CONTINUED)

Run	Date 1973	Q(USGM)	Δ RPM	D	Solids Removal Efficiency, %	% Cake Solids
45	Aug. 30	1.37	32	4	69.8	63.6
46	" 30	1.46	26	4	58.6	65.0
47	" 30	1.42	20	4	63.3	66.2
48	" 30	1.37	12	4	82.2	65.3
49	" 31	1.17	20	4	96.0	43.9
50	" 31	1.08	20	4	95.7	46.8
51	" 31	1.08	20	4	96.0	47.8
52	Sept. 4	1.04	20	2	55.3	63.3
53	" 4	.96	20	4	68.1	66.8
54	" 4	.96	20	4	77.5	67.8
55	" 5	1.00	12	4	61.9	76.6
56	" 5	1.00	12	4	63.9	76.4
57	" 5	1.00	12	4	53.0	77.6
58	" 6	.83	12	4	64.4	52.2
59	" 6	.83	12	4	69.3	51.2
60	" 6	.83	12	4	67.8	42.7 49.0
61	" 7	.79	12	4	63.4	65.6
62	" 7	.79	12	4	57.9	34.3
63	" 11	.83	12	4	55.1	13.8
64	" 11	1.62	12	4	76.8	17.0
65	" 11	.83	12	4	-23.2 D > R	61.6
66	" 11	1.25	12	4	61.0	65.5
67	" 11	1.21	12	4	51.3	72.0
68	" 11	1.59	12	4	49.9	13.8
69	" 12	.83	12	4	56.7	77.8
70	" 12	1.25	12	4	114.3 D > C	75.7
71	" 12	1.67	12	4	69.4	57.7
72	" 14	.79	12	4	57.6	45.0
73	" 14	.92	12	4	56.6	32.9
74	" 17	.83	12	4	51.8	6.1
75	" 17	.83	12	4	46.1	67.4
76	" 18	.88	12	4	62.4	87.0
77	" 18	.90	12	4	68.3	83.2
78	" 18	.91	12	4	62.0	81.0
79	" 18	.89	12	4	66.1	85.9
80	" 18	.87	12	4	48.9	84.4
81	" 18	.82	12	4	51.6	85.5
82	" 18	.93	12	4	59.3	81.2

TABLE A2

RESULTS - SAM 3 DISC CENTRIFUGE

Date	Run	Rotameter %	Flow (USGM)	Flow l/min (3.785 l/gal)	Desludge Time (min)	% Removal	% Sludge	Raw Sewage SS (mg/l)	Centrate SS (mg/l)
Nov.6	1	20	1.0	3.79	10	71.9	0.135	130	37
" 6	2	40	2.0	7.58	10	99.7 D=C	0.17	121	1
" 6	3	20	1.0	3.79	10	81.6	0.13	171	32
" 7	4	13	0.65	2.47	10	101.5 D>C	0.17	166	0
" 7	5	20	1.0	3.79	10	90.0	0.16	163	18
" 7	6	40	2.0	7.58	10	85.1	0.22	166	25
" 7	7	60	3.0	11.36	10	76.8	0.19	148	33
" 7	8	80	4.0	15.15	10	33.8	0.18	78	37
" 7	9	100	5.0	18.95	10	58.1	0.20	139	58
" 7	10	20	1.0	3.79	10	96.4	0.12	127	4
" 8	11	20	1.0	3.79	20	82.1	0.30	216	39
" 8	12	20	1.0	3.79	10	67.3	0.19	197	65
" 8	13	40	2.0	7.58	20	67.3	0.36	192	63
" 8	14	60	3.0	11.36	20	56.6	0.40	187	81
" 8	15	80	4.0	15.15	20	50.6	0.40	161	80
" 8	16	100	5.0	18.95	20	38.1	0.41	155	96
" 8	17	20	1.0	3.79	10	86.2	1.15	2054	283
" 9	18	20	1.0	3.79	20	77.3	-	136	31
" 9	19	20	1.0	3.79	20	65.0	0.26	187	66
" 9	20	20	1.0	3.79	20	76.7	0.31	230	54
" 9	21	20	1.0	3.79	10(part)	90.6	1.16	267	26
" 14	22	20	1.0	3.79	10	63.7	0.49	185	67
" 14	23	40	2.0	7.58	10	68.2	0.78	151	48
" 14	24	68	3.0	11.36	10	77.9	0.88	150	33
" 14	25	80	4.0	15.15	10	53.9	0.98	140	65

TABLE A2 (CONTINUED)

Date	Run	Rotameter %	Flow (USGM)	Flow 1/min 3.785 1/gal	Desludge Time (min)	% Removal	% Sludge	Raw Sewage SS (mg/l)	Centrate SS (mg/l)
Nov.14	26	100	5.0	18.95	10	42.4	1.3	133	76
" 14	27	20	1.0	3.79	20	93.5	0.52	177	11
" 14	28	40	2.0	7.58	20	71.4	1.63	150	43
" 14	29	60	3.0	11.36	20	51.4	1.46	140	68
" 14	30	80	4.0	15.15	20	45.0	1.45	133	73
" 14	31	100	5.0	18.95	20	27.3	1.28	121	88
" 15	32	20	1.0	3.79	20	101.3	0.40	87	0
" 15	33	20	1.0	3.79	20	82.4	0.42	82	14
" 15	34	20	1.0	3.79	20	61.7	0.42	110	42
" 15	35	20	1.0	3.79	20	76.1	0.66	145	35
" 15	36	20	1.0	3.79	20	60.7	0.55	147	58
" 15	37	20	1.0	3.79	20	83.0	0.52	123	21
" 16	38	20	1.0	3.79	20	68.4	0.27	81	25
" 16	39	20	1.0	3.79	20	56.6	0.40	63	27
" 16	40	20	1.0	3.79	20	90.8	0.38	74	7
" 16	41	20	1.0	3.79	20	86.5	0.52	71	9
" 16	42	20	1.0	3.79	20	93.15	0.88	146	10
" 19	43	20	1.0	3.79	20	78.7 avg.	0.23	42	15
" 19	44	20	1.0	3.79	20	68.6	0.33	63	27
" 19	45	20	1.0	3.79	20	71.0	0.40	89	20
" 19	46	20	1.0	3.79	20	101.1	0.37	47	0
" 20	47	20	1.0	3.79	30	84.1	0.46	39	13
" 20	48	20	1.0	3.79	30	83.7	0.52	116	5
" 20	49	20	1.0	3.79	30	84.5	0.97	96	12
" 20	50	20	1.0	3.79	30	77.4	0.92	126	15
" 21	51	20	1.0	3.79	30	87.0 avg.	0.65	122	8
" 22	52	20	1.0	3.79	30	74.3	0.68	129	30
" 22	53	20	1.0	3.79	30	82.8	1.17	141	25
" 22	54	20	1.0	3.79	30	85.2	0.44	136	15
" 22	55	20	1.0	3.79	30	90.0	0.70	131	7
" 22	56	20	1.0	3.79	30	82.7	1.00	140	16
" 22	57	20	1.0	3.79	30	90.0	1.09	136	12
" 23	58	20	1.0	3.79	20	77.0	0.59	132	34
" 23	59	40	2.0	7.58	20	66.4	0.77	124	35
" 23	60	60	3.0	11.36	20	71.3	1.01	184	76
" 23	61	80	4.0	15.15	20	63.2	0.84	128	41
" 23	62	100	5.0	18.95	20	60.2	1.88	148	49

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